Technical Appendix for "Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach"

Olivier Jeanne   Anton Korinek

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1 Derivation of first-order conditions

From the point of an individual, asset holdings could be different from 1. Thus the budget constraints of a domestic consumer are given by

$$\begin{cases} c_0 = d_1 + (1-\theta_1)p_0, \\ c_1 + d_1 = e + d_2 + (\theta_1 - \theta_2)p_1, \\ c_2 + d_2 = \theta_2 y, \end{cases}$$

where θ_t is the quantity of the domestic collateral asset held by the consumer at the beginning of period t and d_t is the debt to be repaid at the beginning of period t. Assuming $\theta_0 = 1$, the symmetric equilibrium implies that $\theta_t = 1$ also for $t = 1, 2$.[1]

2 Numerical illustration: the log-utility-uniform-distribution case

2.1 Preliminaries

If $u(c) = \log c$, then $c^* = 1$. We assume e is uniformly distributed in $[\bar{e}-\varepsilon, \bar{e}+\varepsilon]$. Condition $d_1 < \min e$ is satisfied provided that

$$\varepsilon < \bar{e} - d_1.$$

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[1] In our model consumers are in fact indifferent between asset holdings and bond holdings between periods 1 and 2 so long as they are unconstrained in period 1. As a result, any portfolio of asset/bond holdings such that each consumer consumes c_2 in period 2 constitutes an equilibrium. For simplicity and without loss of generality, we limit our focus on the symmetric equilibrium where the asset and bond holdings of all agents are identical.

Condition (3) is satisfied iff $y < 1$. Then condition (2), $c_1 = m_1 + yc_1$, implies that consumption is given by

$$c_1 = \min\left(1, \frac{m_1}{m^*}\right),$$

where $m_1 = e - d_1$ and we denote

$$m^* \equiv 1 - y > 0.$$

The calibration will be characterized in terms of three parameters: m^*, $\bar{e}$ and ε.

Equation (1) implies that the price of collateral is given by

$$p_1 = y \min\left(1, \frac{m_1}{m^*}\right).$$

First, let us derive a condition that is necessary and sufficient for the economy to be constrained with some probability. If the economy is never constrained then $c_1 = 1$ and condition (4) implies $c_0 = 1$ so that $d_1 = c_0 = 1$. The economy is then indeed unconstrained in period 1 iff $d_2 = c^* + d_1 - e = 2 - e$ is smaller than y for all possible realizations of e, that is if $\bar{e} > 1 + m^* + \varepsilon$. Conversely, if $\bar{e} < 1 + m^* + \varepsilon$ there is a risk that the credit constraint binds in period 1. We will consider calibrations such that the economy would not be constrained in the absence of uncertainty but may be constrained for large negative shocks, that is

$$1 + m^* < \bar{e} < 1 + m^* + \varepsilon.$$

2.2 Laissez-faire

With a uniform distribution for e, the equilibrium condition (4) can be written

$$\begin{aligned}
\frac{1}{d_1} &= E_0[u'(c_1)], \\
&= \frac{1}{2\varepsilon} \int\limits_{\bar{e}-\varepsilon}^{m^*+d_1} \frac{m^*}{e - d_1} de + \frac{1}{2\varepsilon} \int\limits_{m^*+d_1}^{\bar{e}+\varepsilon} de, \\
&= \frac{1}{2\varepsilon} \left[m^* \log\left(\frac{m^*}{\bar{e} - \varepsilon - d_1}\right) + \bar{e} + \varepsilon - m^* - d_1 \right].
\end{aligned}$$

One can show that this equation determines a unique level of d_1, denoted by d_1^{lf}, which satisfies

$$\bar{e} - m^* - \varepsilon < d_1^{lf} < 1.$$

The equation can be solved numerically to find the laissez-faire equilibrium level of d_1 given the exogenous parameters m^*, $\bar{e}$ and ε.

2.3 Social planner

To derive the equations with a social planner note that

$$\lambda_{sp} = \frac{1}{c_1} - 1 = \left(\frac{m^*}{m_1} - 1\right)^+,$$

and that

$$p'(m_1) = \begin{cases} 1/m^* - 1 & \text{if } m_1 < m^*, \\ 0 & \text{if } m_1 \geq m^*. \end{cases}$$

It follows that the social planner first-order condition (equation (5)) can be written

$$\begin{aligned} \frac{1}{d_1} &= E_0[u'(c_1) + \lambda_{sp} p'(m_1)], \\ &= \frac{1}{2\varepsilon} \int_{\bar{e}-\varepsilon}^{m^*+d_1} \left(\frac{1}{e - d_1} + 1 - \frac{1}{m^*}\right) de + \frac{1}{2\varepsilon} \int_{m^*+d_1}^{\bar{e}+\varepsilon} de, \\ &= 1 + \frac{1}{2\varepsilon}\left[\log\left(\frac{m^*}{\bar{e} - \varepsilon - d_1}\right) - 1 - \frac{d_1 - \bar{e} + \varepsilon}{m^*}\right], \end{aligned}$$

which again determines one unique level of d_1, which we denote by d_1^{sp}. One can show that d_1^{sp} satisfies

$$\bar{e} - m^* - \varepsilon < d_1^{sp} < d_1^{lf} < 1.$$

The optimal tax rate on debt inflows is

$$\begin{aligned} \tau &= \frac{E_0\left[\lambda_{sp} \cdot p'(m_1)\right]}{E_0\left[u'(c_1)\right]}, \\ &= \frac{d_1^{sp}(1 - m^*)}{2\varepsilon}\left[\log\left(\frac{m^*}{\bar{e} - \varepsilon - d_1^{sp}}\right) - 1 - \frac{d_1^{sp} - \bar{e} + \varepsilon}{m^*}\right]. \end{aligned}$$

2.4 Calibration

One would like to calibrate the model so as to obtain "reasonable values" for the probability and size of a sudden stop. We now explain how to derive the underlying parameters m^*, $\bar{e}$ and ε from assumptions about the levels of the probability of a sudden stop and of the expected consumption gap $c^* - c_1$ conditional on a sudden stop. The probability of sudden stop is given by

$$\begin{aligned} \pi &= \frac{1}{2\varepsilon} \int_{\bar{e}-\varepsilon}^{m^*+d_1} de, \\ &= \frac{1}{2} - \frac{\bar{e} - m^* - d_1}{2\varepsilon}. \end{aligned}$$

Note that since $d_1 < 1$, we have $d_1 + m^* < \bar{e}$ so that the probability of a sudden stop must be lower that 1/2.

The minimum level of c_1 is given by

$$\begin{aligned} \min c_1 &= \frac{\min e - d_1}{m^*} = \frac{\bar{e} - \varepsilon - d_1}{m^*} \\ &= 1 - 2\frac{\varepsilon\pi}{m^*}, \end{aligned}$$

which implies, if we denote by Δc the expected consumption gap $c^* - c_1$ conditional on a sudden stop,

$$\Delta c = \frac{\varepsilon\pi}{m^*}.$$

Thus from π and Δc we can derive the ratio m^*/ε,

$$\frac{m^*}{\varepsilon} = \frac{\pi}{\Delta c},$$

and

$$\frac{\bar{e} - d_1}{\varepsilon} = 1 + \pi\left(\frac{1}{\Delta c} - 2\right).$$

We can then compute the level of debt under laissez-faire,

$$\frac{1}{d_1} = \frac{m^*}{2\varepsilon}\log\left(\frac{m^*/\varepsilon}{(\bar{e} - d_1)/\varepsilon - 1}\right) + \frac{1}{2} + \frac{\bar{e} - d_1}{2\varepsilon} - \frac{m^*}{2\varepsilon},$$

and substituting out the terms in $(\bar{e} - d_1)/\varepsilon$ and m^*/ε then gives

$$d_1^{lf} = \left[1 - \pi - \frac{\pi}{2\Delta c}\log(1 - 2\Delta c)\right]^{-1}.$$

Given a value for $\bar{e}$, the values of ε and m^* can be derived using

$$\varepsilon = \frac{\bar{e} - d_1^{lf}}{1 + \pi\left(1/\Delta c - 2\right)},$$

$$m^* = \frac{\pi}{\Delta c}\varepsilon.$$

The condition $1 + m^* < \bar{e} < 1 + m^* + \varepsilon$ is satisfied iff

$$\bar{e} > 1 + \frac{\pi}{1 - 2\pi}\frac{1 - d_1^{lf}}{\Delta c}.$$

One can choose $\bar{e}$ arbitrarily subject to this condition.

3 Discussion

Discuss the case where the budget constraint could be binding in period 0. The period-0 price is given by

$$p_0 = E_0\left[\frac{u'(c_1) + \lambda}{u'(c_0)}p_1\right].$$