External Dollar Borrowing in Emerging Markets II:
Volatility, Portfolio Choice, and Amplification

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Abstract

This paper develops a macroeconomic framework of emerging markets that accounts for the mutual endogeneity of the currency composition of external debts and macroeconomic outcomes. Specifically, the level of foreign currency denominated debts affects the volatility of aggregate demand and by extension of the exchange rate. This in turn is an important determinant of the risk premium on local currency debt. Finally, the risk premium is a major factor in borrowers' choices regarding the amount of foreign currency denominated debts.

The mechanism for these relationships is the following: since exchange rates are counter-cyclical, dollar-denominated debts mandate high repayments in low states and vice versa. We show that as a result, external dollar debts amplify economic shocks and raise the volatility of aggregate demand. This in turn increases exchange rate volatility and by extension the risk premium on local currency debt that international lenders charge. Furthermore, these relationships are convex, i.e. the volatility effects of dollar debt exhibit increasing returns to scale, up to a natural limit on dollar debt beyond which macroeconomic volatility diverges. We also establish conditions under which individuals' debt portfolios are socially efficient.

In the described small open economy, the response to exogenous shocks differs substantially from standard models where the amount of dollar debts is exogenous: For example, small increases in international risk aversion entail large amplification effects on macroeconomic volatility as domestic borrowers substitute towards cheaper but riskier forms of debt finance.

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1 Introduction

The analysis of external portfolio holdings has recently received an increasing amount of attention in the open economy macro literature,[1] since a country's asset portfolio affects its response to aggregate shocks. This is particularly important in emerging markets, where a large number of external assets and liabilities are denominated in foreign currencies[2] and where exchange rates are often subject to extreme fluctuations, leading to increased macroeconomic volatility and in extreme cases to financial crises.[3]

At the same time, macroeconomic volatility is an important factor in determining countries' portfolio choices. This implies that is essential for a comprehensive understanding of the macroeconomic dynamics of emerging markets to account for the mutual endogeneity of their external portfolio structure and macroeconomic outcomes such as exchange rate volatility and risk premia.[4]

This paper develops a simple model of the currency denomination of debts in a small open emerging market economy that takes into account the mutual endogeneity of the country's external portfolio structure and of macroeconomic outcomes, specifically the volatility of aggregate demand, exchange rates, and the risk premium on local currency debt. We assume the country has access to two internationally-traded assets, an uncontingent bond denominated in dollars and a contingent bond denominated in local currency.[5]

An integral element of international debt contracts is that one party has to carry exchange rate risk. If the contract is denominated in dollars, the emerging market agent carries this risk, which is counter-cyclical[6] and therefore aggravates the impact of any aggregate shocks in the economy on the agent. If the debt contract is denominated in local currency, international lenders carry the risk and demand a risk premium to compensate them. Naturally, this risk premium is endogenous to the level of macroeconomic volatility.

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[1]Lane and Milesi-Ferretti (2005) and Lane and Shambaugh (2007), for example, analyze the portfolio structure of a large number of countries so as to shed light on the valuation effects of exchange rate shocks. A number of papers such as Gourinchas and Rey (2007) draw inferences for current account adjustment.

[2]Henceforth we will use the standard convention in the literature on "dollarization" to employ the term "dollar debt" for all forms of foreign currency-denominated debts in emerging markets.

[3]Many third-generation models of financial crises (see e.g. Krugman, 1999; Aghion et al., 2004; Schneider and Tornell, 2004, etc.) are based on the feedback effects between dollar-denominated debts and macroeconomic volatility. Also, (Levy Yeyati, 2006) illustrates this point empirically.

[4]In the literature on developed open economies, this point was made by Devereux and Sutherland (2006) and Tille and van Wincoop (2007), who develop DSGE models that endogenize countries' portfolio choices and analyze the implications for macroeconomic adjustment to various shocks.

[5]Uncontingent dollar bond typically play a large role in emerging markets' debt portfolios and have important implications for macroeconomic volatility, as we emphasize below. Local currency-denominated bonds could also be interpreted as any other form of finance that is contingent on the shocks faced by the emerging market economy, i.e. that enhances international risk-sharing.

[6]Calvo and Reinhart (2002) document that emerging markets exhibit "fear of floating," i.e. they attempt to mitigate exchange rate fluctuations so as to reduce the effect of shocks on macroeconomic volatility. However, these attempts are subject to limitations in both directions, as e.g. Argentina and China have experienced in recent years.

In order to jointly determine portfolio positions and macroeconomic outcomes, we proceed in two steps:

1. We analyze the macroeconomic implications of a given debt structure. Since exchange rates are counter-cyclical, dollar-denominated debts entail high repayments in low states when exchange rates depreciate, which aggravates the impact of negative economic shocks, and low repayments in high states when exchange rates appreciate, which amplifies the boom that results from positive economic shocks. Specifically, we show that the greater the fraction of dollar-denominated debts in a country's portfolio, the higher the impact of a given output shock on aggregate demand and the higher consumption macroeconomic volatility. This in turn raises exchange rate volatility and by extension the risk premium on local currency debt that international lenders charge.

   Furthermore, these relationships are convex, i.e. the volatility-enhancing effects of dollar debt exhibit increasing returns to scale: each additional unit of dollar debt raises the variance of the payoff of the existing stock of dollar debt and adds a new unit of dollar debt. Finally, we show that for any given level of total debt, there exists a natural limit for a maximum amount of dollar debt beyond which volatility becomes infinite.

2. We solve for the optimal currency denomination of debts in the economy, taking into account the implications for macroeconomic volatility. When individual borrowers choose the currency composition of their debt portfolio, they weigh the expected costs of fluctuations in consumption that result from dollar debt against any savings from the interest rate spread between the local currency and the dollar. This risk-return tradeoff allows us to describe the amount of dollar debt held in an emerging market economy as an optimal portfolio decision.

The effects of foreign currency debt on macroeconomic volatility constitute externalities. In the next step, we therefore analyze under which conditions private agents choose a socially efficient debt portfolio. Small agents internalize neither the effects of their borrowing choices on exchange rate volatility nor on the risk premium of local currency debt. In equilibrium their borrowing choices are socially efficient only if exchange rates are linear in the output shock: the two pecuniary externalities on exchange rate volatility and on the risk premium offset each other in that case, since security markets are effectively complete.

We also show that the response of a small open economy to exogenous shocks differs significantly when the feedback channels of our model are taken into account: In response to an increase in international risk aversion, the risk premium that international lenders charge on local currency naturally rises; as a result domestic borrowers substitute from local currency towards dollar debt, raising macroeconomic volatility, which in turn raises the risk premium further, leading to an amplification effect. Small changes in international risk aversion can therefore lead to large changes in emerging market volatility. By contrast, increases in the riskiness of the emerging market lead domestic borrowers to take on more local currency debt so as to be less exposed to shocks. As a result, macroeconomic volatility remains virtually constant, while borrowers' expected consumption declines mildly because of the larger interest payments on local currency debt.

The explanations for dollar borrowing in emerging markets in the existing literature have been somewhat unsatisfactory. One difficulty is that in standard models of small open economies with risk-neutral international lenders (see e.g. Obstfeld and Rogoff, 1995), agents who can choose in which currency to denominate their debts would never borrow in foreign currency because they want to avoid exposure to pro-cyclical exchange rate risk.[7]

Most existing papers therefore attribute the phenomenon to some exogenous factor, which typically falls into one of the following two categories: First, some authors emphasize that dollar debt is the only debt instrument available to emerging markets (see e.g. Eichengreen and Hausmann, 2005). This concern, however, has eased as of late; markets for emerging market local currency debt are becoming increasingly liquid (Burger and Warnock, 2006).[8] A second strand of the literature ascribes dollar borrowing to government distortions, such as implicit or explicit bailout guarantees that induce agents to take on excessive risk (see e.g. McKinnon and Pill, 1998; Krugman, 1998; Burnside et al., 2001). This does not seem consistent with empirical evidence that sectors that are more likely to be bailed out actually carry less dollar debt (Eichengreen and Hausmann, 1999).

By contrast, we address the problem by explicitly accounting for the fact that emerging markets face a risk premium on their local currencies, which induces them to take on dollar debt as an optimal portfolio decision. While the precise reasons underlying this risk premium are unclear, Dodd and Spiegel (2005) provide empirical evidence for excess returns on local currency debt in emerging markets. By implication, policies that reduce this risk premium can lead to a decline in dollar borrowing and enhance financial stability.

The remainder of our paper is organized as follows. Section 2 describes the model setup. Section 3 describes the equilibrium of the economy in period 1, i.e. the macroeconomic outcomes for a given amount of dollar debt, and analyzes the implications of the level of dollar debt on volatility. In section 4 we solve for the optimal amount of local currency and dollar debt, analyze the social efficiency of the decentralized equilibrium, and discuss the response of the economy to a number of exogenous shocks, such as an increase in international risk aversion. Section 5 concludes.

[7]A more general analysis of this proposition is given in Korinek (2007b). An exception applies to countries where domestic monetary policy is so unreliable that the volatility of the domestic price level is greater than the volatility of the real exchange rate (Jeanne, 2003). However, in most emerging markets this concern has receded in recent years.

[8]Note that the analysis of the volatility effects of dollar debt in this paper still applies to countries that do not have access to local currency debt, such as some of the least developed countries.

2 Analytical Environment

Let us analyze a small open economy with two time periods denoted $t = 0$ and 1.[9] We assume that there are a continuum of mass 1 of domestic agents and a continuum of international lenders, who are large in comparison to the small open economy. Both are risk averse. Their detailed behavior is described in the following two subsections.

Assume that there are two perishable goods, tradables T and non-tradables N. Tradable goods are the numeraire and can be costlessly moved across borders for international borrowing and lending transactions. Non-tradable goods have to be consumed domestically in the period of their production. We denote the relative price of non-tradables in terms of tradables as p_N, which is a measure for the real exchange rate.[10,11] In period 1, the small open economy experiences an aggregate shock $\omega \in \Omega$, where Ω is the set of all possible realizations.

2.1 Domestic Agents

Domestic agents are born in period 0 with a level of external debt D (denoted in tradable goods). For most applications below, we will assume $D > 0$ so as to capture the situation of emerging markets that are net debtors. In that situation, we can interpret D for example as external borrowing for investment purposes in period 0. However, identical considerations apply for positive levels of wealth, which would correspond to $D < 0$.

Domestic agents derive utility from period-1 consumption C, which is a composite of tradable consumption C_T and non-tradable consumption C_N

$$U = E\left\{\hat{u}(C)\right\} \qquad \text{where} \qquad C = C_T^{\sigma} C_N^{1-\sigma} \tag{1}$$

E is the expectations operator conditional on information available at period 0. $\hat{u}(\cdot)$ is agents' utility function, which satisfies the standard conditions $u'' < 0 < u'$. The

[9] The interpretation of what constitutes one time period in our model depends on the macroeconomic environment of the country under investigation. For a country that is well integrated in the world economy and never faces borrowing constraints, it makes sense to regard macroeconomic variables such as output or external debt in period 1 as the sum of the discounted stream of these variables over the infinite future. On the other hand, for a country that is liquidity-constrained, the values of output and external debt due in a given period are the relevant variables. Most emerging markets are somewhere in between these two polar cases, though the latter case is the one that best describes the situation of a country facing a financial crisis.

[10] More generally, the real exchange rate q is defined as the relative price of a home consumption basket in terms of a foreign consumption basket, i.e. $q = \frac{(p_T)^{\sigma}(p_N)^{1-\sigma}}{(p_T^*)^{\sigma}(p_N^*)^{1-\sigma}}$. If purchasing power parity for tradables is assumed, i.e. $\frac{p_T}{p_T^*} = 1$, the prices of tradable goods cancel out from both baskets. Holding the price of foreign non-tradables constant, it can then be seen that the real exchange rate is a monotonic transformation of the price p_N of home non-tradables relative to tradables.

[11] We employ the given model of real exchange rates for analytical simplicity. The insights of this paper continue to hold for other models of exchange rates, so long as they are counter-cyclical to aggregate demand. This is both an empirical regularity and a property of all common models of exchange rate determination.

parameters σ and $1-\sigma$ are the expenditure shares of tradables and non-tradables in the optimal consumption bundle.[12] β represents their discount factor.

In period 1, domestic agents receive an endowment of tradable and non-tradable goods of $(Y_T^\omega, \bar{Y}_N)$.[13] Tradable output Y_T^ω is subject to an aggregate productivity shock, which can be thought of as either an external terms-of-trade shock (e.g. a neighboring country's devaluation, a fall in the world price of exported commodities etc.) or a domestic slowdown in productivity growth in the export sector. Its expected value is $\bar{Y}_T$.

Domestic agents can borrow or lend abroad.[14] We assume that they have a choice of borrowing in foreign currency debt F, which is linked to the price of tradable goods (the numeraire), or in local currency debt L, linked to the price of non-tradable goods.[15] Foreign currency debt F yields one unit of the tradable good in period 0 and repays a return of R_F units of tradable goods in period 1, which is determined on world markets. The payoffs of local currency debt are linked to the price of non-tradable goods, but all international borrowing and lending transactions have to occur in tradable goods, since non-tradable goods cannot be moved across borders. Local currency thus yields $E[p_N^\omega]$ tradable goods (i.e. the equivalent of one unit of the non-tradable good) and mandates a repayment of $R_L p_N^\omega$ tradable goods (the equivalent of one unit of the non-tradable good plus interest R_L) in period 1.

We assume that there are no limits to how much foreign and local currency individual agents can take on, as long as they maintain strictly positive consumption in all states of the world, i.e. there is no bankruptcy. Furthermore, we assume that the interest rate on local currency debt is greater than on foreign currency debt so as to ensure that the problem has a non-degenerate solution.[16]

[12] The Cobb-Douglas form in which tradable and non-tradable goods enter utility implies that the real exchange rate is a linear function of the output shock. This in turn guarantees that the two assets, dollar debt and local currency debt, span the entire state space and therefore that risk markets are effectively complete for domestic agents. A discussion of the more general case of incomplete risk markets is given below in 4.3.

[13] Analyzing an endowment economy rather than a production economy allows us to focus our analysis on individuals' portfolio decisions. This can be justified as a reasonable approximation of short-run dynamics – in the short run, production factors cannot be re-allocated. Instead, relative prices have to adjust, i.e. the real exchange rate has to appreciate or depreciate. Endogenizing investment would not affect our qualitative results.

[14] Since all agents are identical and risk averse, there is no scope for a domestic credit market.

[15] Literally speaking, this implies that we analyze inflation-indexed bonds in dollars and local currency.

[16] Korinek (2007b) shows that in an economy in which (i) exchange rates are counter-cyclical to aggregate income, (ii) domestic agents can access debt markets in dollars and in local currency and (iii) there is no risk premium on local currency, domestic agents' optimum would entail an infinite amount of local currency debt and infinite holdings of dollar reserves, and this would result in perfect insurance against all aggregate shocks.

We can summarize domestic agents' optimization program as

$$\max_{\{C_T^\omega, C_N^\omega, L\}} E\left\{u([C_T^\omega]^\sigma [C_N^\omega]^{1-\sigma})\right\} \tag{2}$$

$$\text{s.t. } F + E[p_N]L \geq D \tag{3}$$

$$C_T^\omega + p_N^\omega C_N^\omega = Y_T^\omega + p_N^\omega \bar{Y}_N - R_F F - p_N^\omega R_L L \tag{4}$$

2.2 International Lenders

International lenders are large in comparison to the small open economy. By implication equilibrium in international capital markets is exogenous. In the given context, this implies that the economy faces an exogenously given pricing kernel M^ω of international lenders.[17]

For international capital markets to be in equilibrium, the expected return R_i of any asset i with a price distribution $\{p_i^\omega\}$ must obey the pricing condition

$$R_i E[p_i^\omega M^\omega] = E[p_i^\omega] \tag{5}$$

2.3 Definition of Equilibrium

In accordance with the small economy assumption, we take equilibrium in international capital markets as predetermined. Given international lenders' pricing kernel M^ω, an equilibrium in the emerging market economy is defined as

- an allocation $(C_T^\omega, C_N^\omega, F, L)$ and
- prices and returns (p_N^ω, R_F, R_L) for all $\omega \in \Omega$
- which are consistent with international lenders' pricing condition (5)
- which satisfy domestic agents' optimization problem (2)
- and which clear goods markets in all states $\omega \in \Omega$, i.e.

$$\begin{aligned} \text{for tradables in period 0:} \quad & F + E[p_N^\omega]L \geq D \\ \text{and period 1:} \quad & C_T^\omega = Y_T^\omega - R_F F - p_N^\omega R_L L \\ \text{as well as for non-tradables in period 1:} \quad & C_N^\omega = \bar{Y}_N \end{aligned}$$

2.4 Lenders' Equilibrium Returns

In equilibrium, the return on any internationally tradable asset has to satisfy lenders' optimality condition (5). This equation pins down the risk-free return that international lenders demand on dollar debt as $R_F = 1/E[M^\omega]$. From the point of view of the small open economy, lenders' supply of dollar debt is horizontal at that interest rate.

[17] This pricing kernel, or stochastic discount factor, can be derived from any asset pricing model, such as for example a consumption-based asset pricing model.

Let us denote the risk premium that international lenders require for holding local currency debt as ρ, which we define so that

$$(1-\rho)R_L = R_F$$

Substituting this expression into lenders' asset pricing condition $R_L E[p_N^\omega M^\omega] = E[p_N^\omega]$, we find that

$$\rho = -R_F \text{Cov}\left(\frac{p_N^\omega}{E[p_N^\omega]}, M^\omega\right) \tag{6}$$

If the pricing kernel M^ω is constant (risk-neutral lenders) or if exchange rate risk is uncorrelated to M^ω, then $\rho = 0$ and international lenders would supply local currency debt at actuarially fair prices $R_L = R_F$. On the other hand, if lenders are risk-averse (i.e. the pricing kernel is a non-degenerate random variable) and if the real exchange rate p_N^ω is negatively correlated with their pricing kernel, then lenders charge a positive risk premium $\rho > 0$ and the required return on local currency debt is higher than that on dollar debt.

2.5 Domestic Agents in Equilibrium

In competitive equilibrium, representative agents solve optimization problem (2). We can substitute the constraint for tradable consumption C_T^ω and for dollar borrowing F to arrive at the following maximization problem:

$$\max_{C_N^\omega, L} E\left\{\hat{u}\left([Y_T^\omega + p_N^\omega(\bar{Y}_N - C_N^\omega) - R_F D - \ R_L L\left(p_N^\omega - (1-\rho)E[p_N^\omega]\right)]^\sigma [C_N^\omega]^{1-\sigma}\right)\right\} \tag{7}$$

Small agents take the the level of the real exchange rate p_N^ω as well as the expected returns R_F and R_L as given. We obtain the following first-order conditions

$$\text{FOC}(C_N^\omega):\ \hat{u}'(C\omega)\left(\sigma[C_T^\omega]^{\sigma-1}(-p_N^\omega)[C_N^\omega]^{1-\sigma} + [C_T^\omega]^\sigma(1-\sigma)[C_N^\omega]^{-\sigma}\right) = 0$$
$$\text{FOC}(L):\ E\left\{\hat{u}'(C^\omega)\sigma[C_T^\omega]^{\sigma-1}[C_N^\omega]^{1-\sigma} R_L\left[p_N^\omega - (1-\rho)E[p_N^\omega]\right]\right\} = 0$$

In the following subsection, we use the expression $\text{FOC}(C_N^\omega)$, which relates the consumption of non-tradable and tradable goods, to obtain an expression for the real exchange rate in response to the output shock. In section 3 this allows us to characterize equilibrium in the economy in period 1, given a certain choice L of dollar and local currency debt in period 0, and to discuss the response of a small open economy to macroeconomic shocks as a function of private borrowing choices.

The second condition $\text{FOC}(L)$ describes private agents' insurance decision, i.e. the optimal tradoff between costly local currency debt and risky dollar debt in period 0, before the output shock is realized. Using this condition we can solve for the optimal L and for the general equilibrium of the economy in subsection 4.

2.6 Exchange Rate Determination

The first order condition on C_N^ω characterizes equilibrium in the market for non-tradable goods. For the market to clear, non-tradable consumption has to equal the constant non-tradable endowment, $C_N^\omega = \bar{Y}_N$. Changes in aggregate demand thus affect the price rather than the quantity of goods in the non-tradable sector. From the first order condition FOC(C_N^ω), we can obtain the equilibrium level of the real exchange rate

$$p_N^\omega = \frac{1-\sigma}{\sigma} \cdot \frac{C_T^\omega}{\bar{Y}_N} = \psi C_T^\omega \tag{8}$$

The real exchange rate p_N^ω is thus linear in the economy's consumption of tradables C_T^ω, which we can interpret as a measure of aggregate demand. (We define the constant $\psi = \frac{1-\sigma}{\sigma \bar{Y}_N}$ to save on notation.) Positive shocks to productivity in the tradable sector increase the relative availability of tradable goods, which depresses the relative price of tradables and increases the relative price of non-tradable goods. Opposite results hold for negative shocks to the tradable sector.

The linearity of the relationship is a result of the Cobb-Douglas form in which the two goods enter the utility function,[18] but our analysis is robust to a wide range of models of exchange rate determination, as long as exchange rates are counter-cyclical, i.e. they appreciate in times of high aggregate demand and depreciate in times of low aggregate demand. This is both an empirical regularity[19] and characteristic of a wide range of theoretical models of exchange rate determination (see e.g. Obstfeld and Rogoff, 1995). In particular, the real exchange rate modeled in this paper would still be monotonic in aggregate demand for other specifications of the utility function in (1), as long as both tradable and non-tradable goods are normal. More generally, any model of counter-cyclical exchange rates exhibits the property that repayments of dollar debt decline when aggregate demand is high and rise when aggregate demand is low. Furthermore, while we captured the phenomenon of counter-cylical exchange rates here by assuming that productivity shocks occur only in the tradable sector and by emphasizing the response of the real exchange rate, our results apply equally to any other model of aggregate shocks and exchange rate determination that are consistent with exchange rates being counter-cyclical, as is suggested by empirical evidence.

3 Macroeconomic Implications of Dollar Debt

Let us now use the insights from the previous section to analyze the equilibrium of the economy in period 1, given a pre-determined amount of dollar and local currency debt,

[18] Agents spend constant fractions on each good, i.e. $\sigma p_N^\omega C_N^\omega = (1-\sigma) C_T^\omega$. If a positive shock increases tradable consumption, but non-tradable consumption is constant, then the price of non-tradable goods increases such that the value of non-tradable consumption rises in proportion to tradable consumption.

[19] Some emerging markets stabilize their exchange rates out of "fear of floating" (Calvo and Reinhart, 2002). However, as e.g. Argentina or China experienced in recent years, even hard currency pegs are not robust to severe macroeconomic shocks, giving rise to the amplification and volatility effects described in this paper.

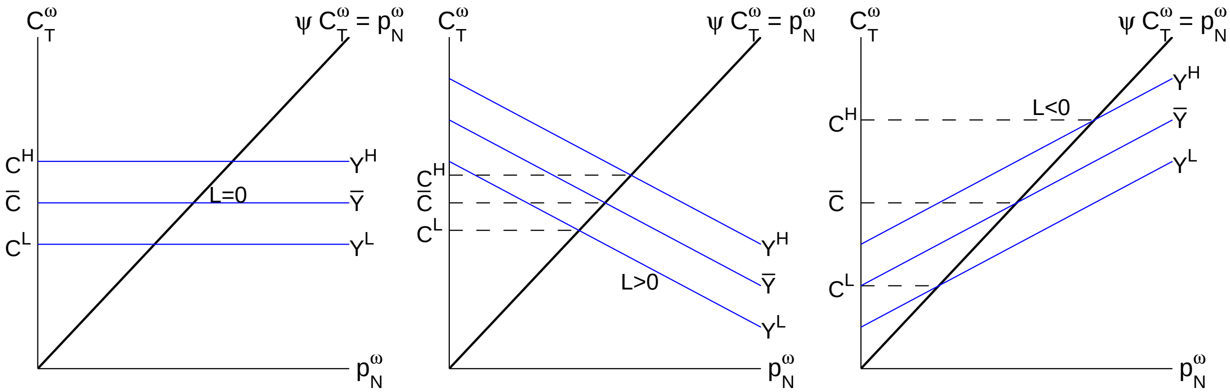

Figure 1: The figure solves for the equilibrium exchange rate and equilibrium level of tradable consumption as a function of several levels of the productivity shock Y^L, $\bar{Y}$, Y^H and of local currency borrowing (i) $L = 0$, (ii) $L > 0$, and (iii) $L < 0$.

as captured by a certain value of L. The market clearing conditions for tradable goods imply that tradable consumption in period 1 has to satisfy

$$C_T^\omega = Y_T^\omega - R_F D - R_L L \left\{ p_N^\omega - (1-\rho) E[p_N^\omega] \right\} \quad (9)$$

where the term in square brackets represents the insurance payoff per unit of local currency. For any given output shock Y_T^ω and an amount of local currency debt L, we can solve the two equations (8) and (9) for the two unknowns C_T^ω and p_N^ω. Recall that we assumed the total level of debt as constant in our analysis, implying that an increase in L represents a shift from dollar to local currency debt within the given stock of debt.[20] A graphical analysis is shown in figure 1.

The left pane illustrates the case without local currency debt. The diagonal $\psi C_T^\omega = p_N^\omega$ represents all combinations of tradable consumption and the real exchange rate for which equilibrium in the non-tradable market prevails according to condition (8). The three horizontal lines labeled Y^L, $\bar{Y}$, Y^H depict equilibrium condition (9) for the tradable sector for three different levels of the output shock Y_T^ω. In the absence of local currency debt, $L = 0$ and the country's entire external debt stock is denominated in foreign currency, i.e. $F = D$. The level of tradable consumption is not affected by the exchange rate; therefore the three lines are horizontal. The level of the output shock Y_T^ω uniquely determines tradable consumption C_T^ω. The real exchange rate adjusts to clear the market for non-tradable goods as given by (8).

The center pane, by contrast, depicts the situation when private agents have taken on a positive amount of local currency debt, i.e. $L > 0$ and $F < D$. When the exchange rate depreciates (low p_N^ω), the repayments on local currency debt are low, implying a higher level of consumption. Conversely, when the exchange rate appreciates, repayments are higher, implying a lower level of consumption. The equilibrium condition (9) for the tradable sector is therefore represented by downward sloping lines. As a result, the impact of productivity shocks (i.e. vertical shifts in this equilibrium condition) on

[20]The level of dollar debt is not bounded at zero; a 'negative amount of dollar debt' would correspond to dollar reserve holdings.

tradable consumption is mitigated: part of the shock is absorbed by the insurance effects of local currency debt; only the remainder shows up in fluctuations in consumption C_T^ω.

Finally, the right pane focuses on the impact of dollar debts coupled with long positions in local currency, i.e. investments that pay off in terms of local currency, implying $L < 0$ and $F > D$. When the exchange rate p_N^ω falls, the value of agents' local currency earnings falls, but they have to repay uncontingent dollar debts. This contracts their consumption. Conversely, when the exchange rate p_N^ω appreciates, their return on local currency holdings rises and they can consume more. The equilibrium condition (9) for the tradable sector is thus represented by upward-sloping lines. In equilibrium, the impact of output shocks Y_T^ω on consumption C_T^ω is amplified by the effects of exchange rate movements on agents' debt burden.

Analytically, we can substitute the exchange rate from (8) into the equilibrium for the tradable goods sector (9) to obtain

$$C_T^\omega = Y_T^\omega - R_F D - \psi R_L L \left[C_T^\omega - (1-\rho)E[C_T^\omega]\right] \tag{10}$$

Naturally, in the absence of local currency debt ($L = 0$), consumption equals production plus initial wealth. For non-zero L, the term in square brackets captures that local currency debt involves selling the risky consumption stream C_T^ω in exchange for lenders' certainty equivalent $(1-\rho)E[C_T^\omega]$. Solving for C_T^ω we find

$$C_T^\omega = \frac{Y_T^\omega - R_F D + \psi R_L L \cdot (1-\rho)E[C_T^\omega]}{1 + \psi R_L L} \qquad \text{where} \qquad E[C_T^\omega] = \frac{\bar{Y}_T - R_F D}{1 + \rho\psi R_L L} \tag{11}$$

where the term $\psi R_L L = \frac{E[p_N^\omega] R_L L}{E[C_T^\omega]}$ can be interpreted as the repayments on local currency debt as a proportion of tradable consumption.

This allows us to analytically derive a number of macroeconomic implications of external dollar debt in emerging markets. Note that our results here describe the equilibrium response of the economy in period 1 to output shocks, given an amount of dollar/local currency debt that was pre-determined in period 0, as captured by a specific value of L. In section 4, we will analyze individual's optimal choice of L.

Proposition 1 (Mitigation/amplification of aggregate shocks) *The higher the share of local currency debt L in an economy, the less the response of consumption C_T^ω to a given output shock Y_T^ω.*

The response of consumption to output shocks is given by

$$\frac{dC_T^\omega}{dY_T^\omega} = \frac{1}{1 + \psi R_L L} \tag{12}$$

This clearly falls in the amount of local currency debt L. Consequently, a given output shock has a smaller impact on consumption the greater the amount of local currency debt in the economy. As we illustrated graphically above, if the economy has taken on local currency debt ($L > 0$) the effect of output shocks on consumption is mitigated.

On the other hand, the more external dollar debt the economy has taken on while investing in local currency ($L < 0$), the more the effect of output shocks is amplified.

In fact, as can be seen from equation (11), we could also express consumption C_T^ω as an affine combination of the output shock plus initial wealth ($Y_T^\omega - R_F D$) and the certainty equivalent of consumption $(1-\rho)E[C_T^\omega]$ with weights α and $(1-\alpha)$, where α is identical to the derivative (12):

$$C_T^\omega = \alpha \cdot (Y_T^\omega - R_F D) + (1-\alpha) \cdot (1-\rho) E[C_T^\omega] \qquad \text{where} \qquad \alpha = \frac{1}{1+\psi R_L L} \tag{13}$$

If $L > 0$, then some of borrowers' debts are denominated in local currency, which moves in parallel with aggregate consumption. This implies that $\alpha \in (0,1)$ and that consumption is a convex combination of $(Y_T^\omega - R_F D)$ and $(1-\rho)E[C_T^\omega]$; therefore output shocks are attenuated. Full insurance ($\alpha = 0$) could only be reached in the limit as $L \to \infty$ (Korinek, 2007b). Intuitively, the reason is that each additional unit of local currency debt reduces exchange rate volatility, and thus reduces the insurance effect of the existing stock of local currency debt – there are decreasing returns to insurance, and no finite amount of local currency debt can result in full insurance in the given framework.

On the other hand, the case $L < 0$ describes emerging market economies that invest in assets that pay off in local currency and borrow in dollars $F > D$ to pay for these. Borrowers effectively leverage their exposure to the domestic output shock: their weight α on the output shock is greater than one, or $dC_T^\omega / dY_T^\omega > 1$. Taking on dollar debt therefore *amplifies* the effects of aggregate fluctuations. Intuitively, a marginal reduction in L raises exchange rate volatility; this raises the destabilizing effect of the existing stock of dollar debt while adding an additional unit of dollar debt. The destabilizing effects of dollar debt are thus subject to increasing returns to scale, up to a natural limit on dollar debt, which will be characterized in proposition 2.

We express also the equilibrium level of consumption in terms of foreign currency debt F rather than local currency debt L:

$$C_T^\omega = \frac{Y_T^\omega - R_F F}{\bar{Y}_T - R_F F} \cdot E[C_T^\omega] \tag{14}$$

The term $Y_T^\omega - R_F F$ captures the actual availability of tradables in the economy after uncontingent debt repayments have been made, whereas $\bar{Y}_T - R_F F$ captures the expected level thereof. Consumption is scaled up or down from its expected level $E[C_T^\omega]$ in parallel to the fraction of these two variables. Analytically, we can characterize the amplification effects of dollar debt by the derivative

$$\frac{dC_T^\omega / E\left[C_T^\omega\right]}{dY_T^\omega / \bar{Y}_T} = \frac{1}{1 - R_F F / \bar{Y}_T} \tag{15}$$

For $F = 0$, i.e. in the absence of foreign currency debt, the output shock and consumption would always increase/decrease by identical fractions; for $F < 0$, i.e. for positive net holdings of foreign currency reserves, the percentage impact of the output shock on consumption is less than one, i.e. the output shock is attenuated; for positive amounts of foreign currency debt $F > 0$ the derivative is greater than one, i.e. the percentage deviation of consumption is greater than the percentage deviation of output caused by the underlying shock.

Proposition 2 (Natural Limit on Dollar Debt) *As $R_F F \to \bar{Y}_T$, the country approaches its natural limit on dollar debt, at which consumption volatility grows to infinity.*

When $R_F F \to \bar{Y}_T$, the country has promised its entire expected tradable production to foreigners in the form of uncontingent dollar debt. From derivative (15) it is easy to see that the response of consumption to the output shock goes towards infinity in that case, i.e. borrowers have infinitely leveraged their exposure to the output shock – the country is 'betting the house.' This corresponds to a long position in local currency debt of $-\psi R_L L \to 1$, which is equivalent to $-p_N^\omega R_L L \to C_T^\omega$. (Recall that negative positions of local currency debt means positive amounts of investments with payoffs denominated in local currency debt.) In other words, any tradable consumption would be financed by the contingent payoffs of local currency reserves, or equivalently, the valuation of local currency reserves is identical the level of tradable consumption.

In the limiting case of $R_F F = \bar{Y}_T$, a below-average realization of the shock to tradable production $Y_T^\omega < \bar{Y}_T = R_F F$ would result in a debt crisis: The country's endowment of tradable goods is insufficient to cover its dollar debts; therefore there is a severe shortage of tradable goods in the economy and the relative price of non-tradables plummets; this implies that the value of local currency bonds falls without bounds, and the returns on local currency investments cannot be used to cover the shortfall of tradable goods.[21] The severe declines in the real exchange rate and in tradable consumption and the parallel rise in the country's current account surplus are reminiscent of the empirical regularities of sudden stops (Calvo et al., 2004).

On the other hand, an above-average shock $Y_T^\omega > \bar{Y}_T = R_F F$ would imply that, after the country repays dollar creditors, there is still a positive amount of tradable production $Y_T^\omega - \bar{Y}_T > 0$ left; tradables are abundant in the economy and the real exchange rate would rise sharply, causing a severe increase in the payoffs on local currency investments and resulting in a strong boom that further appreciates the exchange rate. The (degenerate) equilibrium in such an economy would imply that $p_N^\omega \to \infty$, $C_T^\omega \to \infty$.[22]

The knife-edge case of $Y_T^\omega = \bar{Y}_T = R_F F$ would entail that the country's entire tradable production is precisely sufficient to repay their dollar debts, and that the income on local currency reserves can be consumed. However, there would be a continuum of equilibria: any level of the real exchange rate p_N^ω and corresponding tradable consumption $C_T^\omega = \frac{p_N^\omega}{\psi}$ would fulfil the equilibrium conditions. Whenever consumers raise (reduce) consumption, the real exchange rate appreciates (appreciates) by the same

[21] Algebraically, the (degenerate) equilibrium would entail that $p_N^\omega \to -\infty$, $C_T^\omega \to -\infty$.

[22] The underlying economic dynamics are as follows: consumers increase consumption in an attempt to use up their tradable income $Y_T^\omega - \bar{Y}_T - p_N R_L L$, but in doing so they appreciate the real exchange rate so much that their income from local currency investments $p_N R_L L$ rises by as much as their additional expenditure. Tradable consumption can never catch up with local currency income, implying that the equilibrium is degenerate at ∞.

factor, and the payoffs on local currency investments adjust so as to precisely cover their expenditure on tradable goods.

In panel (ii) of figure 1, the case $\psi R_L L \to -1$ would correspond to the three lines representing equilibria in the market for tradable goods for $Y_T^\omega = Y^H, \bar{Y}, Y^L$ having a positive slope of ψ – identical to the slope of the equilibrium condition for the non-tradable market. It is easy to see that for $Y_T^\omega = \bar{Y}_T$ this results in a continuum of equilibria, whereas for $Y_T^\omega \neq \bar{Y}_T$ no intersection between the two equilibrium conditions exists for finite levels of p_N^ω, C_T^ω.

An important feature of the natural limit to dollar debt is that it is independent of the total amount of debt in the economy. Regardless of the magnitude or sign of the initial level of indebtedness D of the emerging market economy, the total amount of dollar debt is always limited by the expected endowment of tradables $\bar{Y}_T$.

Note that our analysis also provides support for the common argument that an economy's vulnerability to shocks such as sudden stops in capital inflows decreases the more open it is to trade (see e.g. Calvo et al., 2004): the larger the tradable endowment $\bar{Y}_T$ of the economy, the higher the natural limit on dollar debt. Similarly, equation (15) implies that for a given stock of dollar debt F, a larger tradable endowment $\bar{Y}_T$ entails a smaller percentage impact of output shocks on consumption.

Proposition 3 (Consumption and exchange rate volatility) *Consumption volatility and exchange rate volatility fall in the amount of local currency debt L.*

The result on consumption volatility follows directly from the mitigation/amplification of aggregate shocks discussed in proposition 1. The same result applies to exchange rate volatility, since the exchange rate is a linear transformation of tradable consumption. Formally, it follows from equation (11) that

$$\text{Var}(C_T^\omega) = \frac{\text{Var}(Y_T^\omega)}{(1+\psi R_L L)^2} \qquad \text{Var}(p_N^\omega) = \frac{\psi^2 \text{Var}(Y_T^\omega)}{(1+\psi R_L L)^2} \tag{16}$$

It is interesting to note that the marginal insurance effect of local currency debt is declining, i.e. the insurance benefits of local currency debt have decreasing returns to scale: The more local currency debt in the economy, the less the exchange rate fluctuates and hence the lower the marginal insurance effect of an additional unit of local currency. In other words, the functional relationship between the variance of consumption and the amount of local currency debt is decreasing and convex.

By the same token, the more external dollar debt an economy is exposed to, the higher the marginal increase in risk brought on by an additional unit of dollar debt. (Recall that $F = D - E[p_N^\omega]L$, i.e. the two are negatively related.) Each additional unit of dollar debt raises exchange rate volatility and therefore increases the riskiness of the existing stock of dollar debt. When the natural limit on dollar debt as discussed in proposition 2 is reached, consumption and exchange rate volatility become infinite.

Proposition 4 (Risk premium on local currency debt) *The risk premium ρ falls in the amount of local currency debt L.*

The risk premium ρ on local currency debt is a result of lenders' aversion to the country's exchange rate volatility. More insurance in the form of local currency debt L entails more stable exhange rates and therefore a smaller risk premium.[23]

$$\frac{d\rho}{dL} = -\frac{\rho(1-\rho)\psi R_L R_F}{1+\psi R_L L} < 0$$

This has an important implication: the supply of local currency debt is horizontal from the point of view of decentralized agents since they are small. However, for the aggregate small open emerging market economy, the supply of local currency debt is *downward-sloping*, as large lenders adjust the risk premium to the decreasing level of macroeconomic volatility when the aggregate amount of local currency debt to the economy increases.

Proposition 5 (Current account) *The current account balance covaries positively with the output shock if $L > 0$ and negatively if $L < 0$.*

We can write the economy's current account balance as

$$CA^\omega = R_F F + p_N^\omega R_L L$$

In our model, the current account in period 1 simply reflects the repayments on external debt. Note that in the absence of local currency debt, the current account is deterministic since the payoffs of dollar debt are uncontingent. However, when local currency debt is held, the pro-cyclical fluctuations in the real exchange rate and in repayments are also reflected in the current account balance. The covariance of the current account with the output shock is therefore

$$\text{Cov}(CA^\omega, Y_T^\omega) = \text{Cov}(p_N^\omega R_L L, Y_T^\omega) = \psi R_L L \text{Cov}(C_T^\omega, Y_T^\omega) = \frac{\psi R_L L}{1+\psi R_L L}\text{Var}(Y_T^\omega)$$

For positive values of local currency debt L, fluctuations in the current account mitigate the impact of output shocks on consumption: during negative shocks, repayments to foreigners fall and the current account balance is reduced, and conversely for positive shocks. For negative values of L, i.e. high levels of dollar debt, fluctuations in the current account amplify the impact of output shocks. The more foreign currency debt a country has contracted, the more severe the reaction of the current account to output shocks.

Proposition 6 (Expected consumption) *Expected consumption declines as the amount of local currency debt L rises.*

Local currency debt carries a risk premium, i.e. insurance is costly. As indicated by the second part of equation (11) the expected level of tradable consumption equals total expected tradable wealth divided by the term $1+\rho\psi R_L L$, which captures the insurance

[23] A detailed analytical proof of this and the following proposition is provided in appendix A.

premium paid to international lenders. If either $L = 0$ or $\rho = 0$ then expected consumption would equal expected total wealth. If both ρ and L are positive, then the payment of this insurance premium therefore reduces the expected value of consumption.

$$\frac{dE[C_T^\omega]}{dL} = -\frac{\rho\psi R_L R_F E[C_T^\omega]}{1+\psi R_L L} < 0$$

For negative L, domestic borrowers would earn a premium by providing insurance to international lenders.

4 General Equilibrium

4.1 Choice of Currency Denomination

Having characterized the equilibrium in the economy for period 1, given pre-determined levels L and F of local and foreign currency debt, we now turn to solving for the optimal choice of L and F in period 0. Before, however, let us take advantage of the market-clearing condition for the non-tradable sector and the fixed supply of $\bar{Y}_N$ so as to simplify our notation of the utility function $\hat{u}(\cdot)$ into a transformed utility function

$$u(C_T) = \hat{u}(C_T^\sigma C_N^{1-\sigma}) = \hat{u}(C_T^\sigma \bar{Y}_N^{1-\sigma}) \tag{17}$$

Using this notation the first order condition FOC(L) reads as

$$E\left\{u'(C_T^\omega)\left[p_N^\omega - (1-\rho)E[p_N^\omega]\right]\right\} = 0$$

In equilibrium, a borrower's valuation of one unit of local currency with payoff p_N^ω has to equal his valuation of the certainty equivalent, i.e. of the expected value $E[p_N^\omega]$ discounted by lenders' risk premium ρ, where the borrower takes both the distribution of exchange rates $\{p_N^\omega\}$ and the risk premium ρ as given. This can easily be rewritten as the standard insurance condition

$$-\text{Cov}\left\{\frac{u'(C_T^\omega)}{E[u'(C_T^\omega)]}, \frac{p_N^\omega}{E[p_N^\omega]}\right\} = \rho$$

Domestic agents hold local currency debt up to the point where the additional insurance effect per unit of local currency debt, as represented by the covariance term, equals the cost of obtaining the insurance, which is the risk premium on local currency. The higher this risk premium, the less insurance agents take on.

Substituting for the equilibrium exchange rate $p_N^\omega = \psi C_T^\omega$ from (8) and approximating the utility function $u(C_T)$ to the second order yields the condition[24]

$$u''(E[C_T^\omega]) \cdot \text{Var}(C_T^\omega) = \rho E[C_T^\omega] u'(E[C_T^\omega]) \tag{18}$$

[24] Note that if the underlying output shock Y_T^ω is normally distributed, we could apply Stein's lemma to obtain $\text{Cov}(u'(C_T^\omega), C_T^\omega) = E[u''(C_T^\omega)] \cdot \text{Var}(C_T^\omega)$; there would be no need for using an approximation for the covariance term.

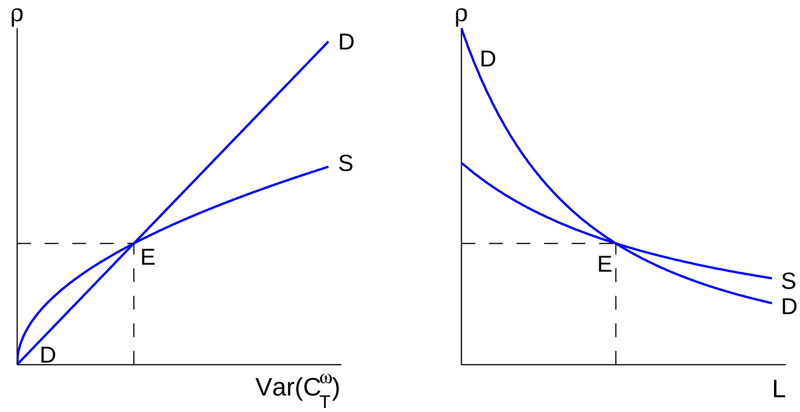

Figure 2: The left panel of the figure depicts the supply locus SS and demand locus DD in a diagram of the risk premium on local currency-denominated debt against consumption volatility in the economy. The right panel shows the same two loci in a standard demand/supply diagram of local currency-denominated debt. The equilibrium E is determined by the intersection of the two curves.

The optimal variance of tradable consumption is directly proportional to the risk premium on local currency debt, i.e. to the cost of insuring against volatility. For a positive risk premium $\rho > 0$ on local currency debt, domestic agents insure imperfectly. They opt for higher consumption volatility the higher the risk premium on local currency debt. In the case $\rho = 0$, the optimal variance of tradable consumption would be zero, i.e. domestic agents would insure perfectly against consumption fluctuations.[25]

4.2 Solution to the Decentralized Equilibrium

The solution to the decentralized equilibrium is given by the 4 equations (6), (8), (10) and (18) in the 4 variables $(\rho, L, p_N^\omega, C_T^\omega)$, of which the last two are state-contingent. Condition (3) then yields the optimal F, and the market clearing condition for non-tradables implies $C_N^\omega = \bar{Y}_N$.

We can collapse the system of four into a system of two equations in two variables, ρ and $\mathrm{Var}(C_T^\omega)$. The first equation is borrowers' optimality condition (18). In the left

[25] As discussed in more detail in Korinek (2007b), this would require an infinite amount of local currency debt and an infinite long position in dollars in the given model. In case the risk premium $\rho < 0$, domestic agents would like to sell all aggregate uncertainty and promise still more contingent payoffs to international lenders. However, this allocation cannot be obtained using the two assets of dollar debt and local currency debt – risk markets are incomplete because agents cannot obtain negative exposure to the domestic output shock. The constrained optimum for borrowers would be to insure perfectly, as in the case of $\rho = 0$. Note that this is an important asymmetry: agents can obtain arbitrary amounts of leverage in the domestic output shock by raising F towards the natural debt limit, but they can reduce their exposure to zero only in the limiting case of $F \to -\infty$, i.e. by holding large amounts of dollar reserves; they can never obtain negative exposure to the shock using debt securities in the two currencies.

For the rest of our analysis, we note that as long as $\rho > 0$, risk markets are effectively complete, i.e. the two instruments of dollar and local currency debt enable agents to attain the first-best equilibrium of complete risk markets.

panel of figure 2, this is depicted as the DD locus, which represents all combinations of consumption volatility and risk premia for which agents' portfolio problem is solved. This locus is approximately linear in ρ.[26]

The second equation can be derived from lenders' supply condition (6), which states that the risk premium that international lenders charge on local currency debt is a function of the covariance of the exchange rate with lenders' pricing kernel. Since the exchange rate is linear in domestic tradable consumption C_T^ω as captured by (8), we obtain

$$\rho = -R_F \frac{\text{Cov}\,(C_T^\omega, M^\omega)}{E[C_T^\omega]} = \xi \cdot \frac{\text{Std}(C_T^\omega)}{E[C_T^\omega]} \tag{19}$$

$$\text{where we define} \quad \xi = -R_F \cdot \frac{\text{Cov}(Y_T^\omega, M^\omega)}{\text{Std}(Y_T^\omega)}. \tag{20}$$

$\xi > 0$ is an exogenous constant that captures how much output fluctuations covary with lenders' pricing kernel, i.e. lenders' aversion to output risk in the emerging economy. The risk premium that lenders require is thus linear in the standard deviation of consumption: the more consumption and the real exchange rate fluctuate, the higher the premium that international lenders require to hold local currency debt. This is international lenders' optimality condition or supply locus of local currency debt. In the left panel of figure 2, the relationship is depicted as SS. The locus SS is concave since the axis in the figure depicts the variance $\text{Var}(C_T^\omega)$, i.e. the square of the standard deviation.[27]

In the left panel of figure 2, the amount of local currency debt (as represented by L) increases as we move from the top right of the figure to the bottom left along both the SS and DD loci, as local currency debt reduces volatility in the economy. In the limit, i.e. as $L \to \infty$, both loci end up at the origin. The linearity and concavity of the two schedules guarantee a unique non-degenerate equilibrium E that pins down the optimal level of the risk premium and of volatility in the economy, and therefore the amount of local currency debt L.[28]

Alternatively, in the right panel of the figure, we show the same two relationships in a traditional demand/supply diagram. Note that the level of consumption volatility depends inversely on the square of the amount of local currency debt in the economy, as given by equation (16); hence the right panel is an inverted version of the left panel. The demand curve DD is approximately a quadratic hyperbola and the supply curve SS a regular hyperbola. Note that supply is downward-sloping because macroeconomic

[26]More precisely, the risk premium ρ enters not only explicitly on the right-hand side of equation (18), but also implicitly in the term $E[C_T^\omega]$: the higher the risk premium, the more borrowers pay for the insurance services of local currency debt, reducing expected consumption. However, for realistic magnitudes, i.e. single digit or low double digit percentages, these indirect effects do not significantly impact the slope of the curve.

[27]Again, the risk premium ρ enters the term $E[C_T^\omega]$ on the left hand side of (19) indirectly, but for typical risk premia the effect on the shape of SS can be neglected.

[28]An alternative way of establishing the uniqueness of equilibrium would be to focus on the formulation of the problem in terms of the share α of output risk that is insured, as discussed on page 12.

volatility and exchange rate volatility falls as L increases; this induces lenders to demand a smaller risk premium, as discussed in proposition 4.[29]

While figure 2 allowed us to determine the optimum amount of local currency debt in the economy graphically, the analytic solution to the two equations (18) and (19) is

$$\mathrm{Var}(C_T^\omega) = \xi^2 \cdot \left(\frac{u'(E[C_T^\omega])}{u''(E[C_T^\omega])}\right)^2 \qquad \rho = \xi^2 \cdot \frac{u'(E[C_T^\omega])}{u''(E[C_T^\omega])E[C_T^\omega]} \tag{21}$$

Both the desired level of consumption volatility and the risk premium that domestic agents are willing to pay are inversely related to their level of absolute risk aversion. Note that these equilibrium conditions are independent of the variance of the underlying output shock. We will use this result and the two graphs developed here in section 4.3 to analyze the effects of changes in the economy's parameter values.

4.3 Optimality of Currency Denomination

When decentralized agents choose in which currency to denominate their debts, they maximize $u(C_T^\omega)$ taking into consideration only how their level of consumption is affected by the amount of insurance they purchase, as captured by condition (9). In doing so, they take the distribution of real exchange rates and the risk premium on local currency as given. On an abstract level, we can thus describe the first-order condition on local currency debt in the competitive equilibrium CE as

$$FOC(L)|_{CE}: \quad E\left\{u'(C_T^\omega) \cdot \frac{\partial C_T^\omega}{\partial L}\right\} = 0$$

The social planner, on the other hand, maximizes welfare in the small open economy as given by the representative agent's utility in (7) while internalizing the general equilibrium effects of her decisions, i.e. her effects on the distribution of real exchange rates $\{p_N^\omega\}$ as given by the additional constraint (8) and on the risk premium of local currency debt ρ as expressed in (6). The social planner (SP)'s first order condition on local currency debt L can be expressed as

$$FOC(L)|_{SP}: \quad E\left\{u'(C_T^\omega) \cdot \frac{dC_T^\omega}{dL}\right\} = 0 \tag{22}$$

where the total differential accounts for the general equilibrium effects of the choice of L on $\{p_N^\omega\}$, R_L and ρ.

Proposition 7 *The decentralized equilibrium in the described economy is socially efficient.*

[29] Since the emerging market economy is small, the traditional effect that supply is upward-sloping because lenders become saturated with emerging market risk is not present. In a model of a 'large' emerging market economy, this traditional effect would counteract the decline in the risk premium that is discussed here.

This result is ultimately an application of the First Fundamental Theorem of Welfare Economics: in the described analytic setup, asset markets are effectively complete, since the Cobb-Douglas form in which tradable and non-tradable goods enter agents' utility function guarantees that the payoffs of local currency debt are linear in the output shock Y_T^ω. The analytic proof is given in appendix A.1.

Intuitively, the decentralized equilibrium is efficient because the pecuniary externalities on the distribution of exchange rates $\{p_N^\omega\}$ and on the risk premium ρ cancel out: Increasing L, i.e. taking on an additional unit of local currency instead of dollar debt, reduces both the volatility of the exchange rate and the level of the risk premium, but it does so in equal proportions. For positive amounts of L, for example, decentralized agents always overestimate the marginal insurance effects of an additional unit of L, since exchange rate volatility and hence the insurance effects of their existing stock of local currency debt fall once all agents increase L. At the optimum, however, they correctly recognize that the marginal insurance effects are zero; therefore the decentralized equilibrium coincides with the social optimum.

We can generalize our analysis to the case of incomplete markets by dropping the assumption that exchange rates are linear in the output shock, as was implied by the Cobb-Douglas consumption index and by the resulting relationship $p_N^\omega = \psi C_T^\omega$. Note that when markets are incomplete, the decentralized equilibrium in an economy with more than one commodity is in general inefficient (Stiglitz, 1982).

Replacing our earlier utility function by a more general function $\hat{u}(C_T^\omega, C_N^\omega)$, the exchange rate would be a non-linear function $p_N(C_T^\omega)$. As long as tradable goods are normal, the exchange rate would still be counter-cyclical, i.e. $p_N'(C_T^\omega) > 0$. The precise conditions when the decentralized equilibrium in such an economy is efficient can be stated as follows.

Proposition 8 *In the incomplete market case, the decentralized equilibrium and the social optimum in the described economy coincide if*

$$E\left\{\frac{u'(C_T^\omega)}{E[u'(C_T^\omega)]}\cdot p_N'(C_T^\omega)\cdot\frac{dC_T^\omega}{dL}\right\} = R_F E\left\{M^\omega p_N'(C_T^\omega)\cdot\frac{dC_T^\omega}{dL}\right\} \tag{23}$$

(See appendix A.2 for a detailed derivation.) $p_N'(C_T^\omega)\cdot\frac{dC_T^\omega}{dL}$ represents the change in the exchange rate in a given state ω that results from an increase in the amount of local currency debt L. Therefore, $R_L L p_N'(C_T^\omega)\cdot\frac{dC_T^\omega}{dL}$ is the resulting change in the repayments on the existing local currency debt stock L.

The condition above basically requires that borrowers and lenders value changes in the distribution of exchange rates that result from altering L equally, i.e. the cost of changes in the payoffs of the existing stock of local currency debt is identical when measured in terms of borrowers' relative utility $\frac{u'(C_T^\omega)}{E[u'(C_T^\omega)]}$ and lenders' relative utility $R_F M^\omega = \frac{M^\omega}{E[M^\omega]}$. If the condition is not satisfied, then the social planner could improve risk sharing between borrower and lender by manipulating L.

To see how such an improvement can be made, let us first express the social planner's optimality condition for L as

$$cE\left[u'(C_T^\omega)\cdot\frac{\partial C_T^\omega}{\partial L}\right]=$$
$$=R_L L\cdot\left\{E\left[\frac{u'(C_T^\omega)}{E[u'(C_T^\omega)]}\cdot p_N'(C_T^\omega)\cdot\frac{dC_T^\omega}{dL}\right]-R_F E\left[M^\omega p_N'(C_T^\omega)\cdot\frac{dC_T^\omega}{dL}\right]\right\} \quad (24)$$

where c is an appropriately defined constant. Note that $E\left[u'(C_T^\omega)\frac{\partial C_T^\omega}{\partial L}\right]$ is downward-sloping. For positive amounts of local currency debt $L>0$, if borrowers value the cost of the changes in the exchange rate distribution (left-hand side of the condition) relatively higher than lenders value the benefits to them (right-hand side), then the social planner would reduce L; this would increase borrowers' utility relatively more than it would hurt lenders and therefore yield a Pareto improvement. Conversely, if the benefit of changes in payoffs for lenders is greater than the relative cost to borrowers, then the social planner would increase L. For negative amounts of L, i.e. for dollar borrowing coupled with external holdings of local currency reserves, domestic agents take the role of lenders and international capital markets that of borrowers; hence the opposite results apply.

Note that in our previous example of linear exchange rates, condition (23) is always satisfied, since $p_N'(C_T^\omega)=\psi$ is constant: this implies that the left-hand side of the condition is reduced to the regular first-order condition $FOC(L)$, which is zero in equilibrium, and the right-hand side is zero as shown in condition (27) in the appendix.

4.4 International Change in Risk-Aversion

External shocks, in particular those emanating from international capital markets, are viewed as having substantial effects on volatility in emerging markets. In this subsection we investigate the effects of a typical shock in this category, an exogenous increase in risk aversion in international capital markets.

Changes in lenders' risk aversion increase the cost at which emerging markets can obtain insurance against domestic shocks, i.e. they raise the rate at which lenders supply funds for a given level of exchange rate volatility, as indicated by the SS equilibrium locus (19). In addition, however, they also entail an important amplification effect: if lenders charge a higher risk premium for a given level of exchange rate volatility, borrowers demand less insurance, as indicated by the DD equilibrium locus (18). This in turn raises consumption and exchange rate volatility in the economy, leading to further rounds of increases in the risk premium on local currency and consumption volatility.

We can model an increase in lenders' risk aversion by performing a mean-preserving spread by a factor α on the pricing kernel M^ω:

$$M_\alpha^\omega=M^\omega+\alpha\cdot(M^\omega-E[M^\omega])$$

We have depicted the case of an increase in risk aversion in figure 3. The direct effect, represented by the horizontal movement from E_1 to the right, is to shift lenders'

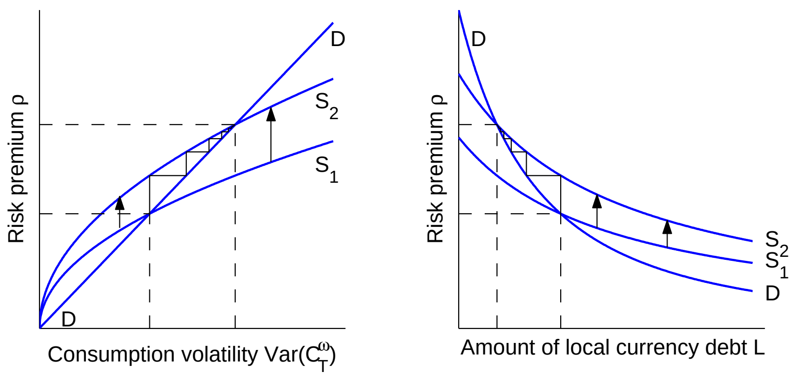

Figure 3: An increase in risk aversion shifts lenders' optimality locus from S_1 up to S_2. This reduces the amount of local currency debt that borrowers contract (right panel) and raises macroeconomic volatility (left panel), thereby amplifying the impact of the shock on the risk premium.

optimality locus SS_1 by the factor α to the right to SS_2: for a given level of exchange rate volatility, lenders demand a risk premium that is by factor α higher than before. However, at lenders' new risk premium, borrowers reduce the amount of local currency that they demand, represented by the vertical upward movement. This results in less insurance and higher consumption and exchange rate volatility. This leads to further rounds of increases in lenders' risk premium and reductions in borrowers' demand for local currency. Taking into account these feedback effects, a small increase in international lenders' risk aversion can lead to much larger increases in risk premia and volatility. When international lenders' risk avesion falls, the opposite effect takes place: risk premia fall and the amount of insurance in the economy rises in a virtuous cycle.

Analytically, an increase in risk aversion by α scales the constant ξ in lenders' optimality condition (19) up by the factor α, i.e. $\frac{d\xi/\xi}{d\alpha} = 1$. The direct effect on the risk premium from the shift of lenders' SS locus (the horizontal movement from E_1 to the right, i.e. holding the amount of insurance L constant) is to increase the risk premium ρ by the same factor:

$$\left.\frac{d\rho/\rho}{\partial\alpha}\right|_{dL=0} = 1$$

We can use the two equilibrium conditions in (21) to express the total effect on the risk premium and output volatility, taking into account general equilibrium effects, i.e. moving to the new equilibrium E_2:

$$\frac{d\rho/\rho}{d\alpha} = 2 \qquad \frac{d\mathrm{Var}(C_T^\omega)/\mathrm{Var}(C_T^\omega)}{d\alpha} = 2$$

In other words, both the risk premium and the variance of consumption increase twice as much as the increase to lenders' risk aversion.

4.5 Changes in the Riskiness of the Domestic Economy

Let us next investigate the effects of an increase in volatility of the output shock in the domestic economy. We model this increase in riskiness by performing a mean-preserving

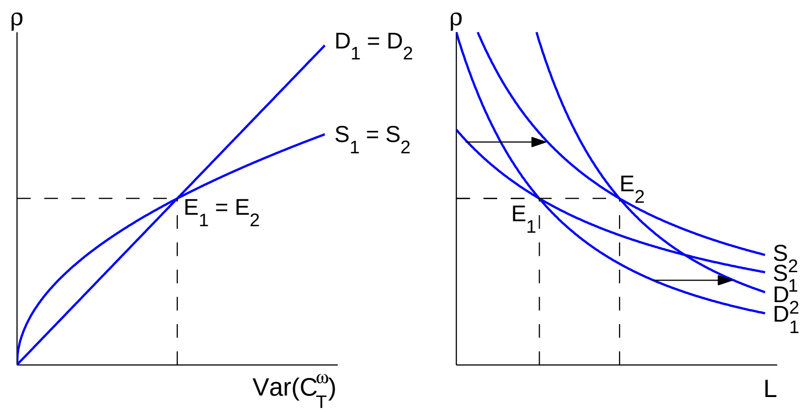

Figure 4: An increase in the volatility of the output shock in the emerging market economy does not change the equilibrium risk premium or consumption volatility as depicted in the left panel. However, the amount of local currency debt required to reach that equilibrium rises, as shown in the right panel.

spread by the factor α on the output shock Y_T^ω, i.e.

$$Y_{T,\alpha}^\omega = Y_T^\omega + \alpha(Y_T^\omega - \bar{Y}_T)$$

However, as a first approximation, the two equilibrium loci DD and SS as given by (18) and (19) are unaffected: both domestic agents and international investors still have the same preferences over volatility in consumption and the real exchange rate. The equilibrium after the change in the volatility of the output shock thus exhibits the same risk premium and the same level of consumption volatility, as depicted in the left panel of figure 4.[30] However, to reach this level of consumption volatility, the equilibrium amount of local currency debt L needs to increase. According to proposition 3,

$$\text{Std}(C_T^\omega) = \frac{\text{Std}(Y_T^\omega)}{1 + \psi R_L L}$$

Naturally, if $\text{Std}(Y_T^\omega)$ is scaled up by a factor of α, then the term $1 + \psi R_L L$ has to grow by the same factor in order to keep consumption volatility and the risk premium constant.

Proposition 9 *An increase in the volatility of the economy's output shock raises the amount of local currency debt that agents incur so as to offset the impact of the higher volatility on consumption.*

The right panel of figure 4 shows the effects of an increase in the volatility of the output shock in a standard diagram that plots demand and supply of local currency debt against its price: both demand and supply curves would shift right by the same

[30] As discussed in the preceding section, there is a small additional effect: the increase in insurance holdings L reduces the expected level of consumption $E[C_T^\omega]$ a little. With decreasing absolute risk aversion, domestic agents would become slightly more risk averse as $E[C_T^\omega]$ falls; hence they would increase the amount of insurance that they take on even further.

factor. For a given price the amount of local currency debt demanded would rise because there is more need for insurance; the supply of local currency debt would rise because macroeconomic volatility increases. The new equilibrium is to the right of the old equilibrium, at an unchanged interest rate.

5 Conclusions

The goal of this paper was to develop a macroeconomic framework that jointly analyzes portfolio decisions and macroeconomic outcomes in emerging markets. One of the key elements of our analysis was aversion to emerging market risk among international investors, which leads to a risk premium on local currency debt.

We showed that dollar-denominated debts increase macroeconomic volatility and by extension risk premia on emerging markets. At the same time, individuals base the currency composition of their portfolios on a tradeoff between the risks of dollar debt and the higher interest rates on local currency debts.

There are a number of questions that are left for future research. Firstly, we performed our analysis in a simple two-period endowment economy so as to focus on a succinct analysis of the feedback channels between country portfolios and macroeconomic outcomes. In a model with multiple time periods, we would have to account (i) for the different maturities of debt flows and production flows and (ii) for individuals smoothing the wealth effects of changes in the valuation of their debts over time. However, in practice both factors lose much of their relevance precisely in those situations in which our analysis is of particular importance, i.e. when emerging markets experience large negative shocks: (i) in such episodes long-term debt markets typically dry up and the average duration of a country's external debt portfolio declines sharply (De la Torre and Schmukler, 2004) and (ii) borrowing constraints tighten and prevent agents from smoothing the impact of valuation shocks over time (Calvo, 1998). The basic model presented here is extended in this direction in Korinek (2007a).

Secondly, we modeled international lenders' aversion to emerging market risk by simply assuming an exogenous pricing kernel that negatively correlates with the aggregate shock in the emerging market economy. While this is sufficient for the insights presented in this paper, a closer examination of the determinants of lenders' risk aversion might yield important insights for policy measures to reduce the risk premium on local currency debt, which would incite emerging market borrowers to take on less risk in their borrowing decisions and would enhance macroeconomic stability. Some potential examples for this are provided e.g. in Eichengreen and Hausmann (2005).

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A Mathematical Appendix

A.1 Optimality Result

Proof of Proposition 7. It is easily verified that the first-order condition $FOC(C_N^\omega)$ is identical for both decentralized agents and the social planner. It remains to be shown that $FOC(L)|_{CE} \Rightarrow FOC(L)|_{SP}$. For a non-degenerate distribution of C_T^ω, this condition is fulfilled if and only if the partial derivative is proportional to the full derivative, i.e. $\frac{\partial C_T^\omega}{\partial L} = c \cdot \frac{dC_T^\omega}{dL}\ \forall \omega$ for some constant c. From conditions 9 and 10 it can be seen that

$$\frac{\partial C_T^\omega}{\partial L} = -R_L\{p_N^\omega - (1-\rho)E[p_N^\omega]\} = -\psi R_L\{C_T^\omega - R_F E[M^\omega C_T^\omega]\} \tag{25}$$

$$\frac{dC_T^\omega}{dL} = -\psi \frac{dR_L L}{dL}\{C_T^\omega - R_F E[M^\omega C_T^\omega]\} - \psi R_L L\left\{\frac{dC_T^\omega}{dL} - R_F E\left[M^\omega \cdot \frac{dC_T^\omega}{dL}\right]\right\} \tag{26}$$

where we used the fact that $(1-\rho)E[C_T^\omega] = R_F E[M^\omega C_T^\omega]$. Multiplying expression (26) by M^ω and taking expectations, we find that

$$E\left[M^\omega \cdot \frac{dC_T^\omega}{dL}\right] = 0 \tag{27}$$

Applying this result back in condition (26) we can show that

$$\frac{dC_T^\omega}{dL} = -\frac{\psi\left(\frac{dR_L}{dL}\cdot L + R_L\right)}{1+\psi R_L L}\cdot\{C_T^\omega - R_F E[M^\omega C_T^\omega]\}$$

and therefore that the full derivative given by equation (26) is proportional to the partial derivative given by equation (25). ■

A.2 Extension to Incomplete Market Case

Proof of Proposition 8. In the incomplete market case, decentralized agents still take exchange rates as given; they perceive that the level of their tradable consumption is determined by expression (9), where the only choice variable is L. The social planner, on the other hand, internalizes all feedback effects to the interest rate and the distribution of exchange rate and recognizes that in general equilibrium, tradable consumption is determined implicitly by

$$C_T^\omega = Y_T^\omega + R_F W - R_L(L)L \cdot \{p_N(C_T^\omega) - R_F E[M^\omega p_N(C_T^\omega)]\}$$

In particular, the social planner internalizes that in the case of a non-linear exchange rate the derivative of the exchange rate $p_N'(C_T^\omega)$ is no longer the constant ψ. The two equations (25) and (26) above therefore change to

$$\frac{\partial C_T^\omega}{\partial L} = -R_L \{p_N(C_T^\omega) - R_F E[M^\omega p_N(C_T^\omega)]\} \tag{28}$$

$$\frac{dC_T^\omega}{dL} = -\frac{dR_L L}{dL} \cdot \frac{1}{R_L} \cdot \frac{\partial C_T^\omega}{\partial L} - R_L L \left\{ p_N'(C_T^\omega) \cdot \frac{dC_T^\omega}{dL} - R_F E\left[M^\omega p_N'(C_T^\omega) \cdot \frac{dC_T^\omega}{dL} \right] \right\} \tag{29}$$

The second term in expression (29) captures the effect of changes in the distribution of exchange rates. If condition (23) is satisfied, i.e. borrowers and lenders value such changes equally, then this term cancels out in the social planner's $FOC(L)$, as given by equation (22), and as a result $FOC(L)|_{CE} \Rightarrow FOC(L)|_{SP}$. ■