Insurance Properties of Local and Foreign Currency Bonds in Small Open Economies

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Abstract

This paper shows that two risk-free bonds, in local and foreign currency, in the limit allow for full insurance in standard small open economy models. Our findings rely on two properties that are common to such models: that exchange rates are counter-cyclical and that international investors are assumed risk-neutral. As a result foreign exchange swaps (i.e. borrowing in local currency and holding foreign reserves) provide a mean zero counter-cyclical payoff that can be used to come arbitrarily close to full insurance against all aggregate shocks. Furthermore, under an imperfectly credible exchange rate peg that collapses in response to large negative shocks, the economy can fully insure against a collapse of the peg. The paper concludes by discussing market frictions that may account for the lack of full insurance in the real world.

JEL Codes: F34, D52, E44
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1 Introduction

A large class of small open economy macro models share two common properties: (i) exchange rates are a decreasing function of aggregate consumption – they appreciate in good times and depreciate in bad times – and (ii) international lenders are risk-neutral towards the small open economy so that interest rate parity between local and foreign currency holds – for example because investors can perfectly diversify the economy's idiosyncratic risks.[1]

Under these conditions, we show that two risk-free bonds, in local and foreign currency, allow the economy to come arbitrarily close to full insurance. In the limit, the two bonds span the entire state space of the small open economy and effectively complete markets. The property that exchange rates are counter-cyclical entails that they always reflect how far aggregate consumption in the economy deviates from its expected steady state value. Therefore foreign exchange swaps (i.e. borrowing in local currency and holding an equivalent amount of foreign currency reserves) in such an economy provide a mean zero counter-cyclical payoff that can be used for insurance purposes. The risk-neutrality of international investors implies that such insurance is actuarially fair and that risk-averse domestic agents find it optimal to take on as much of it as possible, implying full insurance in the limit.

We view the theoretical contribution of our paper as providing a benchmark result: well functioning local and foreign currency bond markets *should* allow for full insurance in small open economies. This theoretical benchmark is clearly at odds with standard empirical evidence: small open economies are *not* perfectly insured against idiosyncratic shocks and are often short in foreign currency (see e.g. Lane and Shambaugh, 2007). Our paper discusses four potential frictions that may be responsible for the lack of insurance in the real world: short-sale constraints, incentive problems, noise in exchange rates and risk aversion in international capital markets.

On a practical level, the paper suggests that well-developed and deep local currency bond markets offer the potential for significant insurance benefits in small open economies. In fact, if local currency bond markets functioned perfectly and exchange rates were always counter-cyclical, additional contingent

[1] The counter-cyclicality of the exchange rate arises in real models of the exchange rate because tradable and non-tradable goods are complements; in monetary models of the exchange rate because money and real goods are complements; in fiscal models of the exchange rate because the tax base is pro-cyclical. For an overview of standard small open economy models see e.g. Obstfeld and Rogoff (1996) or Végh (2010).

assets would be redundant. On the other hand, any factors that reduce the usefulness of local currency bond markets for insurance purposes can entail large welfare costs.

The paper starts out by discussing an economy inhabited by a risk-averse agent that can trade a risk-free bond and claims on its stochastic endowment (i.e. equity) with risk-neutral international investors. Asset markets in such an economy are by construction effectively complete, and equilibrium naturally entails that domestic agents swap the risky endowment against its certainty equivalent. This economy will serve as a reference point for our later results.

In an economy that can trade foreign and local currency bonds and in which the exchange rate is linear in aggregate consumption, a similar situation arises: the linearity implies that foreign exchange swaps span the same state space as an equity security and exhibit similar insurance properties. However, aggregate consumption and by extension exchange rates are endogenous to how much insurance the economy has taken on through swaps: the better insured, the less aggregate income and consumption flucuate, the less the exchange rate fluctuates, and the less insurance an additional unit of the swap provides. In the limit, as the economy's holdings of the swap go towards infinity, aggregate consumption will be perfectly insured against endowment shocks and the exchange rate will be constant.

Our main result is that these findings can be generalized to an economy in which the exchange rate is any arbitrary non-linear decreasing function of aggregate consumption. In such an economy foreign exchange swaps and equity generally span different state spaces for any finite amount of swaps. However, the payoffs provided by foreign exchange swaps are still strictly countercylical, and aggregate consumption in the economy becomes smoother the more of the swap it has taken on, implying less insurance in a given state the closer that state is to full insurance. We show that in the limit, the economy still achieves the same equilibrium as in the previous two cases, and risk markets are effectively complete: aggregate consumption can be perfectly insured against endowment shocks and the exchange rate will be constant.

Assume for example a state of the world in which consumption declines far below its expected value; the exchange rate depreciates strongly, and a foreign exchange swap provides a large insurance payout; this payout in turn moderates the decline in aggregate consumption and so the exchange rate depreciates less in that state. In the limit, as the amount of foreign exchange swaps in the economy goes to infinity, aggregate consumption in each state asymptotically approaches its expected level, and the country is fully insured against aggregate fluctuations. Our full insurance result also extends to exchange rates that fluctuate within target zones.

In economies with an exchange rate peg, foreign exchange swaps cannot be used to insure against macroeconomic fluctuations that leave the exchange rate unchanged. However, if the currency peg is imperfectly credible and collapses in response to large negative shocks, the economy can fully insure against such a collapse of the peg by taking on large amounts of foreign exchange swaps. As a result the collapse of the peg would never actually occur.

The paper first illustrates our results using a real model of a two-period endowment economy, in which the exchange rate is represented by the relative price of non-tradabe in terms of tradable goods. Next we extend our setup to an infinite horizon dynamic stochastic general equilibrium framework and show that the full insurance result continues to hold in the limit. Furthermore, the insurance properties of local and foreign currency bonds are unchanged if we introduce persistent endowment shocks and capital accumulation in such a model. In order to underline the generality of our results, the appendix sketches a standard monetary model as well as a fiscal model of exchange rate determination and demonstrates that both satisfy the countercyclicality condition that is required for our insurance results to hold.

This paper is related to the literature on financial dollarization, which focuses on potential reasons for the use of foreign versus local currency debt, and the associated macroeconomic risks in small open economies, esp. emerging market economies (see e.g. Ize and Levy Yeyati, 1998; Jeanne, 2003; Eichengreen and Hausmann, 2005, among many others). Relative to this literature, the novelity of our paper is to analytically investigate the insurance capabilities and spanning properties of foreign and local currency bonds when the distribution of exchange rates and portfolio choice are solved for jointly.

Our paper contributes to the literature on the insurance benefits of different financial instruments for small open economies that face incomplete risk markets (see e.g. Borensztein et al., 2004). One strand of this literature focuses on instruments that are directly linked to the income shocks that an economy faces, such as GDP- or commodity price-indexed bonds (see e.g.

Froot et al., 1989; Shiller, 1993). The implicit assumption is that agents could best reduce their consumption risk by insuring against shocks to income. This paper focuses on an alternative approach: if the exchange rate of a country is strictly counter-cyclical to aggregate consumption, as suggested by standard small open economy macro models, then the exchange rate constitutes an excellent statistic of the level of consumption in the economy. By extension, insurance instruments that are contingent on the level of the exchange rate, such as foreign exchange swaps, allow indirectly for insurance against shocks to consumption. The contribution of this paper is to show that under the conditions embodied by standard small open economy models, such insurance instruments can in theory fully insure a small open economy against aggregate shocks.[2]

The paper is also closely related to Ching and Devereux (2000) who investigate the special case of an economy in which the exchange rate is a linear function of consumption: in such a setup local currency bonds naturally span the entire state space and can be used to achieve full insurance as a limiting result. Our paper show that this insurance result is more general and holds for arbitrary non-linear exchange rate functions, as long as the exchange rate is a strictly monotonic function of aggregate consumption.

Geanakoplos and Kubler (2003) study in an incomplete markets setting whether dollar or local currency bonds allow for better risk-sharing between a developed and a developing country. We focus instead on how the combination of the two bonds can in the limit effectively complete risk markets for a small open economy. In a similar vein, Angeletos (2002) and Buera and Nicolini (2004) show that governments in a closed economy can combine uncontingent bonds of different maturity to effectively complete risk markets.

The insurance channel in our paper works through counter-cyclical exchange rate fluctuations and is distinct from the terms-of-trade movements that provide insurance in large country models with multiple traded goods, as those described in Cole and Obstfeld (1991) and a number of recent papers on international portfolio holdings such as Engel and Matsumoto (2009) and

[2] In practice, few forms of indexation are perfect, and the relative desirability of insurance instruments that are linked to an economy's income shocks versus instruments that are linked to an economy's level of consumption – such as the exchange rate – is an empirical matter. Magill and Quinzii (1995), for example, show that the relative insurance benefits of an imperfectly indexed real bond and a nominal bond with uncertain purchasing power depend on how close each bond comes to a security of constant purchasing power.

Coeurdacier et al. (2009). In small open economy models like ours, domestic shocks by definition do not affect the terms of trade, but they do affect exchange rates, allowing them to play an insurance role.[3]

2 A Simple Insurance Framework

We start out by introducing a simple insurance framework of a small open economy, which serves as a benchmark for our analysis of local currency bonds below. The economy consists of two time periods 0 and 1 and has one homogenous and tradable consumption good. We assume that there are two sets of agents, international lenders and a representative domestic agent. International lenders are large in comparison to the small open economy, do not discount future income and are risk-neutral.[4]

The representative domestic agent has a utility function $u(C)$ over period 1-consumption that satisfies $u''(C) < 0 < u'(C)$ and the Inada conditions. In period 1, the agent is subject to an aggregate endowment shock $A^\omega \in [A^{\min}, A^{\max}] \subset \Re^+$ that depends on the state of nature $\omega \in \Omega$ and has an expected value $E[A^\omega]$. In period 0, the domestic agent has no initial wealth but can trade two securities with international lenders, a risk-free bond with zero interest and a risky asset that represents a claim on the domestic economy's endowment, i.e. that yields $E[A^\omega]$ in period 0 and requires a contingent repayment of A^ω in period 1. The risky asset can be interpreted for example as equity such as FDI, or as a GDP-linked security. Given this structure, risk markets are effectively complete for the domestic agent. We denote his holdings of risk-free bonds by F and of the risky asset by α.

The agent's period 0 budget constraint restricts his portfolio choice to bundles (F, α) that satisfy $F + \alpha \cdot E[A^\omega] = 0$. The resulting level of aggregate income in period 1 is $Y^\omega = (1-\alpha)A^\omega - F$. Using these two constraints, the domestic agent's problem is to maximize expected utility

$$\max_\alpha E\left[u\left((1-\alpha)A^\omega + \alpha E[A^\omega]\right)\right] \tag{1}$$

[3]Furthermore, exogenous terms-of-trade shocks to a small open economy can be interpreted as shocks to the economy's endowment of tradable goods.

[4]Our results continue to hold under the weaker assumption that international lenders can perfectly diversify the small open economy's risk, as is common in the small open economy literature.

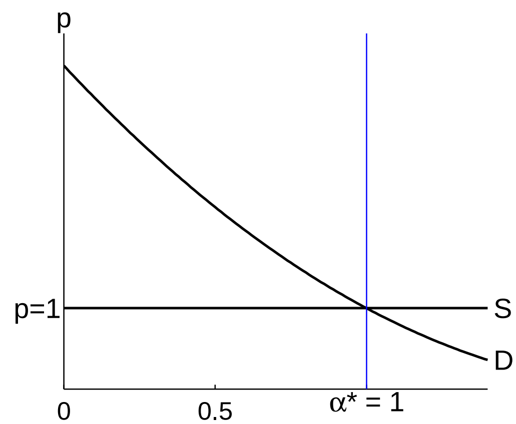

Figure 1: Standard insurance equilibrium with risk-neutral insurer

Given the assumptions on the agent's utility function, his optimal asset holdings are trivially $\alpha = 1$ and $F = -E[A^\omega]$. In other words, the agent swaps the entire risky income stream A^ω against the certainty equivalent $E[A^\omega]$ and fully insures against the aggregate productivity shock.

Proposition 1 *A risk averse representative agent in a small open economy who can trade a risk-free asset and a GDP-linked security with risk-neutral international investors will obtain full insurance.*

The result is depicted graphically in figure 1. The supply S of international lenders is perfectly elastic since they are risk-neutral and trade the risky asset at its expected value $E[A^\omega]$. Decentralized agents are risk-averse: the more of the risky asset they hold, the less they are willing to pay for it, as indicated by the downward-sloping demand curve D. At $\alpha^* = 1$, they are fully divested of risk.

While the solution to the discussed insurance problem is almost trivial, we would like to point out the following feature: A general result in Arrow-Debreu economies is that n securities are required to fully insure against risk across n states of the world. The example just presented is a special case: in an economy with one risk-neutral and one risk-averse agent, two securities – a risk-free bond and a security that represents a claim on the risky endowment of the risk-averse agent – suffice to effectively complete risk markets and obtain optimal risk-sharing. The risk-averse agent offloads all his risk by selling claims on his risky endowment to the risk-neutral agent in return for a fixed payoff of equal expected value.

In the following section we show that this result can be extended to a setup where a risk-averse agent can trade a risk-free bond together with a security offering payoffs that are a strictly monotonic function of his income. An important practical example of such a setup are models of small open economies that trade bonds denominated in dollars and local currency. In most common small open economy models with flexible exchange rates, the exchange rate is a strictly monotonic function of aggregate consumption. In the following section 3 we formally derive our results using a model of real exchange rates. Appendix A illustrates that exchange rates are also counter-cyclical to aggregate consumption for monetary and fiscal models of exchange rate determination.

3 Local Currency Debt As Insurance

To develop a simple model of an exchange rate, we extend our model economy of the previous section to two goods, a tradable and a non-tradable. The relative price p_N of the two goods constitutes a measure for the real exchange rate of the economy.[5] We assume that the domestic agent receives a stochastic endowment A_T^ω of the tradable good in period 1 and that his endowment of the non-tradable good is, for simplicity, fixed at $\bar{A}_N = 1$.[6] He derives utility from consuming the two goods according to the function

[5] This notation follows a common convention in the theoretical literature to capture the real exchange rate by the relative price of non-tradable in terms of tradable goods. In this setup, a rise in p_N constitutes an appreciation and a fall in p_N reflects a depreciation in the real exchange rate. By contrast, the real exchange rate itself is typically defined as the price of a foreign consumption basket in terms of a home consumption basket. Note that the two measures move inversely, i.e. a strictly counter-cyclical real exchange rate implies a strictly pro-cyclical price of non-tradable goods $p_N'(\cdot) > 0$. The counter-cyclical real exchange rate in this section could easily be converted into a counter-cyclical nominal exchange rate, e.g. by assuming that there exists a central bank that sets the nominal rate E so as to stabilize a consumer price index $P(E, p_N)$ that comprises tradable goods that are priced in world markets and non-tradable goods with relative price p_N.

[6] It is well known that shocks to the endowment of non-tradable goods cannot be insured since such goods cannot be transported across borders (see e.g. Cole and Obstfeld, 1991). Our analysis therefore only applies to shocks in the tradable sector. The same results hold for models without a non-tradable sector, such as the monetary and fiscal models of exchange rate determination that are discussed in appendix A.

$u(C_T, C_N)$, which is twice continuously differentiable, quasiconcave, satisfies the Inada conditions, and in which both goods are ordinary.

The asset market of the economy consists of two real bonds: one repays a unit of the tradable good in period 1 and represents foreign currency debt, the other repays one unit of the non-tradable good in period 1 and represents local currency debt. Even though non-tradable goods – by definition – cannot be moved across borders, foreign investors can trade financial assets denominated in non-tradable goods and convert the proceedings into tradable goods at the prevailing exchange rate p_N^ω. Lenders pay $E[p_N^\omega]$ tradable goods for one unit of local currency debt in period 0 and require a state-contingent repayment of p_N^ω tradable goods in period 1. We denote the representative domestic agent's holdings of local and foreign currency debt by L and F, and we formulate his budget constraint for period 0 and his tradable income for period 1 as:

$$\begin{aligned} F + LE[p_N^\omega] &= 0 \\ Y_T^\omega &= A_T^\omega - F - Lp_N^\omega \end{aligned} \tag{2}$$

We eliminate $F = -LE[p_N^\omega]$ by combining the two equations. Observe that increasing the amount of local currency debt L requires a reduction in foreign currency debt F (i.e. an increase in foreign reserves $-F$) by the amount $E[p_N^\omega]$. Changing L therefore performs the same function as buying a foreign exchange swap: the agent swaps the risky payoff p_N^ω against the certain payoff $E[p_N^\omega]$. The Lagrangian of the representative agent's optimization problem is

$$\mathcal{L} = E\left\{u\left(C_T^\omega, C_N^\omega\right) - \lambda^\omega\left[C_T^\omega + p_N^\omega C_N^\omega - A_T^\omega - p_N^\omega \bar{A}_N + L(p_N^\omega - E[p_N^\omega])\right]\right\}$$

Equilibrium is characterized by a choice of local and foreign currency debt (L, F), a vector of consumption allocations (C_T^ω, C_N^ω) and a vector of real exchange rates (p_N^ω) that satisfy the agent's optimization problem and budget constraints and clear markets for every state of the world ω. In the non-tradable sector market clearing requires $C_N^\omega = \bar{A}_N$ and in the tradable sector $C_T^\omega = Y_T^\omega\ \forall\omega$ as determined by equation (2).

Denoting $\partial u(\cdot)/\partial C_T = u_T(\cdot)$ and similarly for non-tradable goods, the first-order conditions are:

$$\begin{aligned} \text{FOC}(Y_T^\omega) &: u_T(\cdot) = \lambda^\omega \\ \text{FOC}(Y_N^\omega) &: u_N(\cdot) = \lambda^\omega p_N^\omega \\ \text{FOC}(L) &: E\left\{\lambda^\omega \cdot (p_N^\omega - E[p_N^\omega])\right\} = 0 \end{aligned} \tag{3}$$

Combining the first two conditions pins down the real exchange rate

$$p_N^\omega = \frac{u_N(C_T^\omega, C_N^\omega)}{u_T(C_T^\omega, C_N^\omega)} = p_N(C_T^\omega) \tag{4}$$

which is a function of aggregate tradable consumption C_T^ω only, as non-tradable consumption is a constant. (In our simple model, aggregate tradable income equals aggregate consumption or aggregate demand for tradables, i.e. $Y_T^\omega = C_T^\omega$, and we can use the three terms interchangably.) Since both goods are ordinary, the real exchange rate is a strictly increasing function, i.e. $p_N'(Y_T^\omega) > 0$. Equation (3) can be transformed to read as

$$\mathrm{Cov}(u_T(\cdot), p_N^\omega) = 0$$

Aggregate tradable consumption $C_T^\omega = Y_T^\omega$ and the real exchange rate $p_N(C_T^\omega)$ are defined by the system of equations (2) and (4), which expresses the mutual dependence of the two variables: the income and consumption of the representative agent depend on the exchange rate, which determines the level of repayments, and the exchange rate depends on the representative agent's level of consumption through standard equilibrium effects. We combine the two conditions into the following implicit equation, which depends on the endowment shock A_T^ω and the chosen level of local currency debt L:

$$C_T^\omega = A_T^\omega - L\left\{p_N(Y_T^\omega) - E[p_N(C_T^\omega)]\right\} \tag{5}$$

The agent's tradable consumption consists of endowment income minus the payoffs of the foreign exchange swap. We restrict L to the interval $(-A_T^{\min}/E[p_N^\omega], +\infty]$. This rules out degenerate levels of consumption, which could arise if the agent takes on an amount of foreign currency debt $F > A_T^{\min}$ that exceeds the minimum tradable endowment of the economy and that could therefore lead to bankruptcy for low realizations of A_T^ω. Since we assumed the representative agent's utility function satisfies the Inada conditions, this restriction is always satisfied in equilibrium.

Example of Linear Exchange Rates

We first consider a simplified example where the two goods enter in Cobb-Douglas form into the utility function, $u(C_T, C_N) = f\left(C_T^{\frac{1}{1+\sigma}} C_N^{\frac{\sigma}{1+\sigma}}\right)$ with $f(\cdot)$

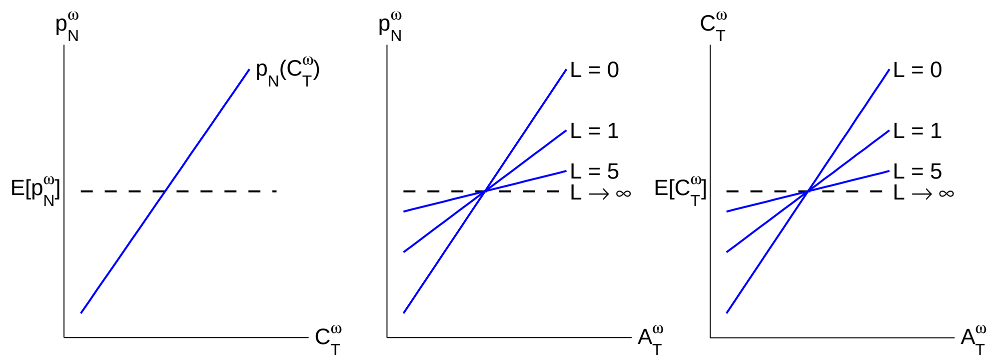

Figure 2: Insurance function of local currency debt: the case of linear exchange rates

strictly increasing and $\sigma > 0$. Under these assumptions, the real exchange rate takes the linear form $p_N = \sigma C_T^\omega$.[7]

Given investors' risk neutrality, the expected value of the domestic agent's tradable consumption is always $E[Y_T^\omega] = E[A_T^\omega]$ regardless of his asset allocation. For the linear exchange rate specification, aggregate tradable consumption from equation (5) can be solved explicitly,

$$C_T^\omega = \frac{A_T^\omega + \sigma L E[A_T^\omega]}{1 + \sigma L} \tag{6}$$

Tradable consumption is a weighted average of the risky output shock A_T^ω and the certainty equivalent $E[A_T^\omega]$. As the amount of local currency debt (or foreign exchange swaps) in this expression rises, the relative weight on the expected value increases. In the limit $\lim_{L\to\infty} C_T^\omega = E[A_T^\omega]$ tradable consumption is constant and the agent is fully insured.

It is easy to see that this problem is isomorphic to the insurance problem we analyzed in the preceding section. The two expressions for aggregate income in the two sections, equations (1) and (6), coincide if we set $\alpha = \frac{\sigma L}{1+\sigma L}$. The span of the shock A^ω in that section and of local currency debt p_N^ω in the current section coincide for linear specifications of the exchange rate. The solution to the problem with a linear exchange rate is thus the limit result $(F^*, L^*) = (-\infty, \infty)$ – at the optimum the agent takes on an infinite amount of local currency debt and holds an infinite amount of foreign currency reserves; this provides him with full insurance.

[7] Ching and Devereux (2000) employ a similar model of linear exchange rates that is derived from a money-in-the-utility function setup.

Figure 2 illustrates our findings graphically. In the left panel, the real exchange rate is depicted as a stable linear function of aggregate consumption. The center panel shows that the exchange rate is also linear in the endowment shock A_Y^ω so that local currency debt spans the same state space as the endowment shock. By taking on insurance in the form of foreign currency swaps L, the agent receives insurance against the output shock; hence the right panel shows that aggregate consumption C_T^ω becomes less and less sensitive to the endowment shock as L increases. By the same token, the exchange rate, which is a function of aggregate consumption, becomes less sensitive to the endowment shock (center panel). As $L \to \infty$, aggregate consumption and the exchange rate become constant, implying full insurance and no exchange rate fluctuations.

General Case

Let us now analyze the general case: if $p_N(C_T^\omega)$ is a non-linear function of aggregate consumption, the endowment shock A_T^ω and local currency debt p_N^ω span different subspaces for any finite L. Nonetheless, optimizing agents still reach the same first-best equilibrium as in the previous section as a limiting result:

Proposition 2 *Assume a small open economy that (i) has counter-cyclical exchange rates, i.e.* $p'_N(C_T^\omega) > 0$ *and (ii) can trade local and foreign currency bonds with risk-neutral international investors. A representative agent in this economy will in the limit obtain full insurance against aggregate shocks.*

Proof. We prove the proposition in two steps:

1. The ordering of tradable consumption (and by extension of the exchange rate) across different state is unaffected by the agent's portfolio choice of (F, L), i.e. for any two ω_1 and ω_2 with $A_T^{\omega_1} < A_T^{\omega_2}$ it holds that $C_T^{\omega_1} < C_T^{\omega_2}$ and $p_N^{\omega_1} < p_N^{\omega_2}$.

   To see this, we keep L constant while taking the total differential of equation (5). This implies

   $$\left.\frac{dC_T^\omega}{dA_T^\omega}\right|_L = \frac{1}{1 + L \cdot p'_N(C_T^\omega)} > 0$$

   where the inequality follows from our assumption that $p'_N(\cdot) > 0$ and the lower bound on L. In the limit as $L \to \infty$, consumption no longer

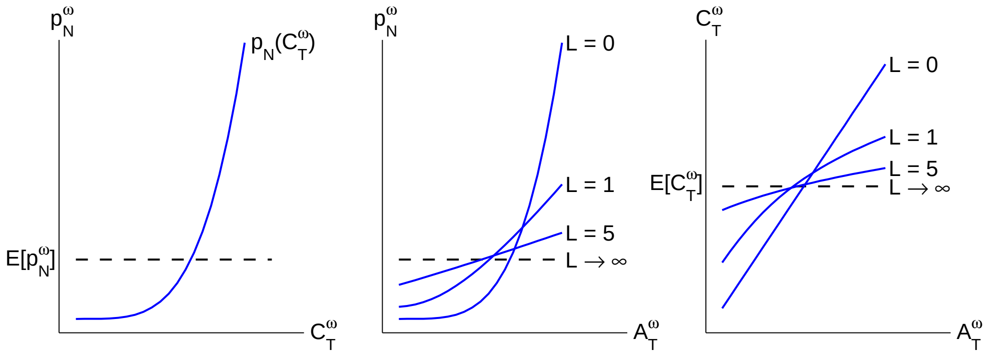

Figure 3: Insurance function of local currency debt: general case

depends on the output shock because all the risk is swapped away:

$$\lim_{L\to\infty} \left.\frac{dC_T^\omega}{dA_T^\omega}\right|_L = 0$$

Since the expected payoff of the swap is zero, this constant level of consumption has to equal the expected value of output,

$$\lim_{L\to\infty} C_T^\omega = E[A_T^\omega] \quad \text{and} \quad \lim_{L\to\infty} p_N^\omega = p_N(E[A_T^\omega])$$

2. The agent's first-order condition of L implies that the marginal utility of buying more of the swap (increasing L) is positive for any finite L:

$$\frac{dE[u(C_T^\omega, \bar{Y}_N)]}{dL} = -E[u_T(\cdot)\cdot(p_N^\omega - E[p_N^\omega])] = -\text{Cov}(u_T(\cdot), p_N^\omega) > 0$$

For any finite L, $A_T^{\omega_1} < A_T^{\omega_2}$ implies that $C_T^{\omega_1} < C_T^{\omega_2}$ and $p_N^{\omega_1} < p_N^{\omega_2}$ according to step 1. Since $u_T(\cdot)$ is strictly decreasing, the covariance above is negative, and the agent can increase his utility by increasing L. As a result, he strives to buy an unbounded amount of the swap, and the optimum is the limiting result $(F^*, L^*) = (-\infty, +\infty)$. According to step 1 this leads to a constant level of consumption in the limit, i.e. to perfect insurance against the endowment shock.

■

At an intuitive level, the result can be described as follows: each unit of the foreign exchange swap provides for a mean zero insurance payment

$(p_N^\omega - E[p_N^\omega])$. By the monotonicity of the exchange rate function $p_N(\cdot)$, this insurance payment is larger the more aggregate consumption C_T^ω surpasses its expected level $E[C_T^\omega]$. The state-contingent payment therefore reduces aggregate consumption in high states and raises it in low states. The closer consumption in a given state comes to its average level $E[C_T^\omega]$, the more the exchange rate in that state approaches its expected value $E[p_N^\omega]$ and the smaller (in absolute value) the insurance payment. In the limit, aggregate consumption is fully insured and the exchange rate is constant at $p(E[A_T^\omega])$.

Figure 3 illustrates these mechanics for an exemplary convex real exchange rate function $p_N(C_T^\omega)$. At $L = 0$, aggregate income $C_T^\omega = A_T^\omega$ so that $C_T(A_T^\omega; L = 0)$ is a linear function (right panel); the curvature of $p_N(A_T^\omega) = p_N \circ C_T(A_T^\omega; L = 0)$ is identical to that of $p_N(C_T^\omega)$ (center panel); the payoff of the swap $p_N^\omega - E[p_N^\omega]$ is convex in C_T^ω and by extension in A_T^ω. This implies that the swap initially provides (in absolute terms) more insurance in high states when the real exchange rate strongly appreciates than in low states when it mildly depreciates.

Increasing L makes aggregate consumption $C_T(A_T^\omega; L)$ concave in the productivity shock (right panel): consumption deviates less from its mean in high states than in low states; the exchange rate function $p_N(A_T^\omega) = p_N \circ C_T(A_T^\omega; L)$ becomes less convex and resembles more and more a linear function (center panel). Ultimately, the closer consumption C_T^ω in a given state comes to the expected value $E[A_T^\omega]$, the smaller the marginal insurance effect of the swap $p_N(C_T^\omega) - E[p_N(C_T^\omega)]$ in that state. In the limit of $L \to \infty$, aggregate tradable consumption is fully insured across all states.

In the described model with two assets but a potentially infinite number of states, the non-linearity of the exchange rate implies that risk markets are incomplete for any finite value of L. However, as $L \to \infty$, the exchange rate spans the same space as the endowment shock A_T^ω and the representative agent can obtain full insurance as a limiting solution to his optimization problem. We thus conclude that risk markets become effectively complete in the limit.

Target Zones

The result of the above proposition can be generalized further: agents in a small open economy will obtain full insurance in the limiting case $L \to \infty$ even for weaker conditions on the function $p_N(C_T^\omega)$. A sufficient condition is, for example, that (1) there exists a neighborhood around $E[C_T^\omega] = E[A_T^\omega]$

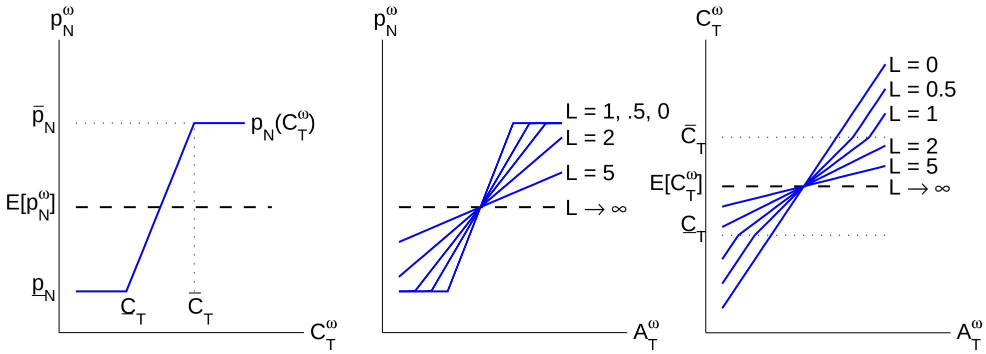

Figure 4: Insurance function of local currency debt with target zone for exchange rate

in which the function is differentiable and strictly increasing in income, i.e. $\exists \epsilon > 0 : p'_N(C_T) > 0 \; \forall \; C_T \in [E(C_T^\omega) \pm \epsilon]$, and (2) that the exchange rate is more depreciated than average $p^\omega > p(E[C^\omega])$ for below-average realizations of consumption $C^\omega < E[C^\omega]$ and more appreciated $p^\omega < p(E[C^\omega])$ for above-average realizations $C^\omega > E[C^\omega]$.

Condition (2) guarantees that the payoffs of the swap are positive in lower-than-average output states and negative in higher-than-average output states, which implies that C_T^ω will eventually be moved into the described neighborhood of $E[A_T^\omega]$ for any ω, even if the exchange rate function outside the neighborhood is not monotonic or continuous. In other words, for any extreme realization A^ω, there is an $\hat{L}(A^\omega)$ so that $C(A^\omega; L)$ will be in the described neighborhood for any $L \geq \hat{L}(A^\omega)$. Once enough insurance has been taken on to move consumption across all states into this neighborhood, condition (1) guarantees that endowment fluctuations are further smoothed out as was described in proposition 2.

A practical example is the case of a government that stabilizes the exchange rate within a target zone, as illustrated in figure 4: Exchange rate fluctuations are cut off at the top and bottom, i.e. the exchange rate cannot move outside of the interval $[\underline{p}_N, \bar{p}_N]$. Let us denote the lower and upper thresholds of aggregate tradable consumption where these limits are reached as $\underline{C}_T$ and $\bar{C}_T$.

A simple mechanism for implementing such a policy in the given model is to impose taxes and subsidies on tradable consumption, which are rebated in a lump sum fashion to neutralize any income effects.[8] Assume for example that aggregate income falls short of the lower threshold $\underline{C}_T$ in a given state ω. Then a tax τ^ω on tradable consumption would introduce a wedge $(1+\tau^\omega)$ into the marginal rate of substitution between tradable and non-tradable goods in equation (4) so that

$$\hat{p}_N(C_T^\omega, \tau^\omega) = \frac{u_N(C_T^\omega, \bar{A}_N)}{u_T(C_T^\omega, \bar{A}_N)} \cdot (1+\tau^\omega) \tag{7}$$

The tax (subsidy) rate τ^ω can be chosen so as to maintain a given real exchange rate target $\hat{p}_N(C_T^\omega, \tau^\omega) = \underline{p}_N$ or $\bar{p}_N$. In such an economy, the neighborhood condition discussed above is always satisfied, and the local currency swap provides counter-cyclical insurance in all states of the world, as illustrated in figure 4.

Let us pick a state ω in which the tradable endowment falls short of the lower threshold $\underline{C}_T$. Increasing L provides at first a positive payoff $-\left(\underline{p}_N - E[p_N^\omega]\right)$, which raises the agent's tradable income. At some point, the agent's income in state ω surpasses the cutoff $\underline{C}_T$ (the lower dotted line in the graph). From this point onwards, $p_N'(C_T^\omega) > 0$ and the results of the previous proposition apply. The limiting equilibrium entails $L \to \infty$ and full insurance.

4 Exchange Rate Pegs

The previous section analyzed the insurance capabilities of local currency debt assuming that exchange rates are flexible, at least in a neighborhood around the average level of the endowment shock. If countries peg their exchange rate – for reasons such as e.g. "fear of floating" (Calvo and Reinhart, 2002) – the full insurance result of the previous section can no longer be achieved.

However, a common feature of most exchange rate pegs is that they are not perfectly credible, or at least not at all times. As for instance the Argentine crisis of 2001/02 illustrated, even countries with a currency board –

[8] Our results on the insurance properties of local currency debt under a target zone regime are independent of precisely how and why the target zone is implemented. Providing a more comprehensive welfare-based foundation for target zones is beyond the scope of this article. See e.g. Krugman and Miller (1993) for further details.

the strongest possible institutional commitment to a fixed exchange rate – are prone to depreciations of their currency when they are hit by sufficiently strong negative shocks. Similarly, a peg at an undervalued exchange rate is often not fully sustainable in case of strongly positive shocks, as China experienced in 2005 when it started to revalue then yuan.

When exchange rate pegs are imperfectly credible, local currency debt can still serve as an insurance instrument against states when the peg falls, but it cannot smooth consumption in normal times when the exchange rate remains pegged. As in the previous section, we model the mechanism through which government pegs the real exchange rate as a tax or subsidy on tradable income. We assume that for incentive reasons or for political economy reasons, the maximum tax rate that the government can impose on tradable income is $\bar{\tau}$. If the endowment shock A_T^ω is low enough and this tax ceiling becomes binding, the real exchange rate p_N^ω will depreciate to[9]

$$\hat{p}_N(C_T^\omega, \bar{\tau}) = \frac{u_N(C_T^\omega, \bar{A}_N)}{u_T(C_T^\omega, \bar{A}_N)} \cdot (1 + \bar{\tau}) < \bar{p}_N$$

We can then find the following analogon to proposition 2 for the case of imperfectly credible exchange rate pegs:

Proposition 3 *Assume a small open economy that has (i) an exchange rate peg that collapses when aggregate consumption falls below a threshold* $\bar{C}_T < E[A^\omega]$*, i.e.* $p_N^\omega = \bar{p}_N$ *for* $C_T^\omega \geq \bar{C}_T$ *and* $p_N^\omega = \hat{p}_N(C_T^\omega, \bar{\tau}) < \bar{p}_N$ *and strictly increasing for* $C_T^\omega < \bar{C}_T$ *and (ii) that can trade local and foreign currency bonds with risk-neutral international investors. A representative agent in this economy will in the limit obtain full insurance against declines in consumption below the threshold* $\bar{C}_T$*, and the exchange rate will never depreciate below* $\bar{p}_N$*.*

[9]This paper does not focus on the question of when or why exchange rate pegs might be an optimal policy (see Garber and Svensson, 1995; Calvo and Reinhart, 2002, for a discussion); instead we take the existence of pegs as given and focus on the implications for the insurance characteristics of local currency debt. Another potential mechanism to model imperfectly credible pegs would be to assume that the government has a buffer R of tradable goods that it uses to stabilize the agent's tradable income Y_T^ω and the level of the exchange rate. In such a model, the peg would collapse if the buffer is insufficient to restore consumption to the expected level. See also our discussion of fiscal models of the exchange rate in the appendix for a further potential motivation for a collapsing peg.

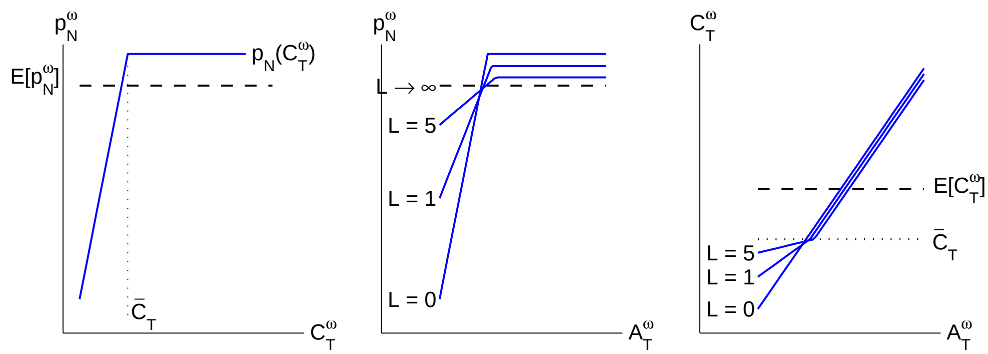

Figure 5: Insurance function of local currency debt under an imperfectly credible peg

The proof follows along the same lines of the proof of proposition 1. However, local currency debt can only insure against those realizations of consumption in which the peg collapses and the exchange rate depreciates, i.e. for $C_T^\omega < \bar{C}_T$ in our example. Increasing local currency debt L, i.e. buying local currency swaps, reduces tradable income and consumption in states in which the peg is maintained, but provides positive payoffs in those (low) states in which the peg collapses.

As in the previous section, the optimal amount of exposure to local currency debt is infinite. In the limit, tradable income is fully insured against declines below the threshold $\bar{C}_T$ and the exchange rate never actually depreciates, i.e. $\lim_{L\to\infty} \inf_{\omega\in\Omega} C_T^\omega = \bar{C}_T$ and $\lim_{L\to\infty} \inf_{\omega\in\Omega} p_N(C_T^\omega) = \bar{p}_N$. In other words, under the stated assumptions, a peg that we initially assumed to be imperfectly credible can always be maintained.

Figure 5 illustrates these results graphically: the exchange rate peg collapses for realizations of tradable consumption below $C_T^\omega < \bar{C}_T$ (left panel). A local currency swap can therefore insure against income falling below that level (right panel): As $L \to \infty$, tradable consumption in low productivity states is stabilized at $\bar{C}_T$ and no more exchange rate depreciations take place (center panel).

5 Infinite Horizon

This section demonstrates that our results carry over into a standard infinite horizon dynamic stochastic general equilibrium environment. We keep our notation from earlier sections but add a time subscript t to all variables. Assume the discount factor of domestic agents to be $\beta < 1$ and the expected return that international investors demand on foreign and local currency loans to equal r which satisfies $\beta(1+r) = 1$. Furthermore, to make our analysis more interesting, suppose that the endowment shock in the small open economy follows an AR(1) process with innovations ϵ_t that are distributed i.i.d. over the interval $[0, \infty)$.

$$A_t^\omega = \rho A_{t-1}^\omega + (1-\rho)\bar{A} + \epsilon_t^\omega \tag{8}$$

For expositional convenience, we describe the economy's portfolio choice in terms of foreign currency bonds F_t and foreign exchange swaps S_t – this is a simple transformation of the choice between foreign and local currency bonds, since a foreign exchange swap consists of a short position in local currency bonds and a long position in foreign currency bonds of identical magnitude.[10]

Bond market of any maturity at time 0

We first analyze the case in which there is a period 0 market for bonds and swaps (or, equivalently, local and foreign currency bonds) maturing at any horizon $t \geq 1$. The economy's initial parameters are without loss of generality $F_0 = 0$ and $A_0 = \bar{A}$, and the stochastic stream of endowments starts in period 1 according to equation (8). We denote by F_t and S_t the agent's choice of foreign currency bonds and swaps maturing at time t. The maximization problem of a representative agent in that economy is

$$\max_{\{C_{T,t}^\omega, C_{N,t}^\omega, F_t, S_t\}} \sum_{t=1}^{\infty} \beta^t u\left(C_{T,t}^\omega, C_{N,t}^\omega\right) \quad \text{s.t.} \quad \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^t F_t = 0$$
$$\text{and} \quad C_{T,t}^\omega + p_{N,t}^\omega C_{N,t}^\omega = A_{T,t}^\omega + p_{N,t}^\omega \bar{A}_N - F_t - S_t\{p_{N,t}^\omega - E[p_{N,t}^\omega]\} \ \forall t, \omega$$

We set up a Lagrangian to this problem with current value multipliers λ_0 and λ_t^ω on the constraints, take the first-order conditions and simplify. The intra-period trade-offs of tradable and non-tradable consumption yield an expression for the shadow value $\lambda_t^\omega = u_T(\cdot)$ and the familiar exchange rate

[10] In other words, we could re-write any portfolio of foreign currency bonds F_t and swaps S_t as a portfolio of foreign and local currency bonds $(\hat{F}_t, \hat{L}_t)$ such that $\hat{F}_t = F_t - S_t E[p_{N,t}^\omega]$ and $\hat{L}_t = S_t$.

condition $p^{\omega}_{N,t} = p_N(C^{\omega}_{T,t})$ as in equation (4), which now holds for every state and time period. The intertemporal optimality conditions imply

$$FOC(F_t) : \lambda_0 = E[\lambda^{\omega}_t] = E[u_T(C^{\omega}_{T,t}, \bar{A}_N)] \ \forall t \tag{9}$$

$$FOC(S_t) : E\{\lambda^{\omega}_t(p^{\omega}_{N,t} - E[p_{N,t}])\} \ \forall t \tag{10}$$

The first of these conditions requires that consumption will in expectation be perfectly smoothed over time. Since expected output is the same in all time periods, this implies $F_t = 0 \ \forall t$. The second condition is analogous to condition (3) and can be read as $\mathrm{Cov}(u_T(\cdot), p_N) = 0$. We can therefore apply the proof of proposition 2 to every time period of the model economy and find that it is optimal for domestic agents to take on an infinite amount of the local currency swap for every time period $S_t \to \infty \ \forall t$. As before, this will entail full insurance against domestic consumption risk as a limiting result. The intuition is, as before, that any deviation of consumption from its expected value will lead to a deviation of the exchange rate from its expected value, which implies a non-zero payoff to the foreign exchange swap and returns the economy to the full insurance equilibrium.

One-period bonds

We obtain identical results for a market setup in which only one-period bonds in local and foreign currency exist. However, this setup will add a new twist to the results of the previous sections. Given the one-period nature of bonds, it is most convenient to write the Lagrangian of domestic agents in recursive form

$$\begin{aligned} \max_{\{C_{T,t}, C_{N,t}, F_{t+1}, S_{t+1}\}} V(F_t, S_t; A_{T,t}) &= u(C_{T,t}, C_{N,t}) - \\ - \lambda_t \left[C_{T,t} + p_{N,t}C_{N,t} - A_{T,t} - p_{N,t}\bar{A}_N + F_t - \frac{F_{t+1}}{1+r} + S_t(p_{N,t} - E[p_{N,t}]) \right] &+ \\ + \beta E V(F_{t+1}, S_{t+1}; A_{T,t+1}) & \end{aligned} \tag{11}$$

It can easily be verified that the first-order conditions and envelope conditions to this maximization problem lead to the same optimality conditions as in the case of a period 0 market for bonds maturing in every period and that the economy will arrive at the same full insurance allocation as was described there, involving $S_t \to \infty \ \forall t$. However, the bond holdings F_t through which the economy achieves this full insurance outcome are different from what was described before. The reason for this arises from the following problem: (1) shocks to the endowment process $A^{\omega}_{T,t}$ are persistent, but (2) the economy can earn non-zero insurance payoffs only in the period that a shock hits, not in later periods, as international lenders demand the same expected return on local and foreign currency bonds in every period.

When the tradable endowment process of the economy is shocked by an amount $\epsilon_t = \Delta$, the endowment will in expectation be $\rho\Delta$ above steady state next period, $\rho^2\Delta$ the period after, and so on. This implies a present discounted value of the endowment shock of $\frac{\Delta}{1-\frac{\rho}{1+r}} = \frac{1+r}{1+r-\rho}\Delta$. In the absence of insurance instruments, an optimizing domestic agent who obtains such an endowment shock would smooth his spending over time by saving the entire endowment shock and increasing his consumption only by a fraction $r\frac{1+r}{1+r-\rho}\Delta$ every period. However, as a result of the increase in consumption, the exchange rate would appreciate. This is how the payoffs of a currency swap in period t alone can insure the agent against a persistent shock that lasts for many periods.

A domestic agent's level of consumption in period t will be above steady state after such a shock as long as the present discounted value of his wealth is increased. Increased consumption implies an appreciated exchange rate and positive payouts to international investors on the foreign exchange swap, which will reduce the positive impact of the shock and lead the agent to accumulate debt F_{t+1}. The agent's level of consumption will ultimately return to steady state once he has accumulated an amount of debt $F_{t+1} = \rho\frac{1+r}{1+r-\rho}\Delta$ – this is precisely the present value of the remainder of the shock starting next period. In other words, the foreign exchange swap completely neutralizes the effect of the shock on the agent's lifetime wealth and, by extension, insures the agent perfectly against persistent endowment shocks. Abstracting from any further shocks, the economy's endowment converges back to $\bar{A}_T$ over time and the agent's debt is slowly paid off using the excess of the endowment over its steady state level $A_{T,t} - \bar{A}_T$.

6 Investment

This section expands our results from an endowment economy to an economy with investment. By the same argument as in the previous section, we will show that foreign currency swaps can in the limit provide full insurance against the wealth effects caused by productivity shocks. As a result, the

Fisherian separation theorem will hold and investment choices in such an economy will be efficient, given the cost of capital dictated by international investors.

We denote the domestic economy's level of capital K_t, which depreciates at a rate δ, and which can be augmented by converting tradable consumption goods into capital at a rate of one-to-one. Capital is employed using the neoclassical production function $A^{\omega}_{T,t}G(K_t)$, which is premultiplied by the stochastic productivity parameter $A^{\omega}_{T,t}$. We add this component to the Bellman equation of the previous section

$$\begin{aligned}\max_{\{C_{T,t},C_{N,t},K_{t+1},F_{t+1},S_{t+1}\}} & V(K_t, F_t, S_t; A_{T,t}) = u(C_{T,t}, C_{N,t}) - \\ & - \lambda_t[C_{T,t} + p_{N,t}C_{N,t} - A_{T,t}G(K_t) - p_{N,t}\bar{A}_N + (1-\delta)K_t - K_{t+1} + F_t - \\ & \quad - F_{t+1}/(1+r) + S_t(p_{N,t} - E[p_{N,t}])] + \beta EV(K_{t+1}, F_{t+1}, S_{t+1}; A_{T,t+1})\end{aligned}$$

This setup adds the following optimality condition to the conditions obtained in the previous section,

$$\begin{aligned}1 &= E\left\{\frac{\beta u_T(C^{\omega}_{T,t+1})}{u_T(C_{T,t})} \cdot [A^{\omega}_{T,t+1}G'(K_{t+1}) + (1-\delta)]\right\} = \\ &= \frac{E[A^{\omega}_{T,t+1}]G'(K_{t+1}) + (1-\delta)}{1+r} + \text{Cov}\left(\frac{\beta u_T(C^{\omega}_{T,t+1})}{u_T(C_{T,t})}, A^{\omega}_{T,t+1}G'(K_{t+1})\right)\end{aligned} \tag{12}$$

where we used the Euler condition on bond holdings (9) to obtain the second line. Note that the covariance term in this expression is negative as long as the representative consumer is imperfectly insured against productivity shocks, since periods of high productivity coincide with low marginal utility of consumption. This captures that a risk-averse representative investor invests less in risky projects than a risk-neutral investor.

However, the first-order condition on local currency swaps (10) implies – as before – that the agent takes on an arbitrarily large position in the swap so as to come arbitrarily close to full insurance. In the limit, the covariance term in equation (12) turns zero. The remainder of the optimality condition (12) can than be transformed to yield the standard neoclassical optimality condition for capital investment with risk-neutral financiers, and Fisherian separation is restored:

$$E[A^{\omega}_{T,t+1}]G'(K_{t+1}) + (1-\delta) = 1 + r$$

The intuition for why the economy can achieve both full insurance and productive efficiency in the presence of auto-correlated productivity shocks is as before: A positive productivity shock leads the representative domestic agent to borrow from abroad and increase the capital stock, while simultaneously raising his consumption as a reflection of the increased net present value of his income. This latter effect leads to a rise in the exchange rate and a negative payoff on the foreign exchange swap position. Equilibrium is restored at the point where the negative payoff from the swap precisely offsets the positive effects of the productivity shock – i.e. where the shock is borne entirely by international investors and the domestic agent is fully insured.

Similar results hold for any shock that affects the wealth – and by extension consumption – of the representative agent, including anticipated productivity shocks or investment productivity shocks.

7 Extensions

In the previous sections we showed that local currency debt can – in theory – perform a close-to-perfect insurance function against fluctuations in aggregate income and in exchange rates under a regime of flexible exchange rates, and against collapses in pegs under pegged exchange rates. These theoretical predictions hold in any standard open economy macro model in which exchange rates are counter-cyclical and international lenders are risk-neutral. However, our results stand in marked contrast to empirical findings: in practice, many small open economies are exposed to a significant amount of net foreign currency debt (Lane and Shambaugh, 2007) and are not insured against aggregate risks – often such risks are even amplified by international capital flows (Kaminsky et al., 2005; Aguiar and Gopinath, 2007).

This section discusses a number of market frictions that may be responsible for the lack of full insurance in the real world, in particular (i) short-sale constraints, (ii) incentive problems, (iii) noise in exchange rates and (iv) risk-averse international capital markets. This discussion provides guidance for how to adjust standard small open economy macro models to allow for a non-degenerate analysis of questions of debt denomination and insurance.

Short-Sale Constraints

The full insurance results in the previous two sections required that $L \to \infty$ and $F \to -\infty$, i.e. that agents take on infinite amounts of local currency debt and foreign currency reserves. If we interpret L strictly as debt, then this is problematic, since a short-sale constraint on L is likely to bind at some point.

However, it can be argued that the sophisticated financial engineering tools in today's markets offer direct swaps between currencies that limit counterparty risk to the net payoff of the swap and significantly relax short-sale constraints (recall that $|p_N^\omega - E[p_N^\omega]| \cdot L < |A_T^\omega - E[A_T^\omega] \forall \omega$ for $L > 0$). This should allow small open economies to come quite close to the full insurance equilibrium. Furthermore, while the existence of short-sale constraints would explain why borrowing in local currency debt is limited, it cannot explain why many economies in fact take on dollar-denominated debt. Consequently, imposing a short-sale constraint on an agent's asset positions is unlikely to solve the puzzle presented by our findings.

Incentive Problems

Another common argument for why small open economies (and in particular emerging market economies) borrow little in local currency debt relate to a moral hazard problem: there is a concern that local currency debts create an incentive for domestic governments to erode the value of the currency and thereby expropriate international lenders (see e.g. Tirole, 2003). However, government actions to erode the value of the currency are typically counter-cyclical: devaluations or spending-induced inflation usually occur in bad times, whereas governments behave more responsibly in good times. As a result, government actions to manipulate the exchange rate in response to output shocks in fact enhance the insurance function of local currency debt. As long as the government's actions are performed in a predictable manner, lenders would simply adjust the interest rate on local currency so as to offset any expected losses, and decentralized agents would still be willing to borrow in local currency. However, if exchange rate manipulations are uncorrelated to shocks to aggregate income, then this introduces noise into the exchange rate, which reduces the insurance capabilities of local currency debt. We discuss this case further in the ensuing subsection.

Noise in Exchange Rates

While exchange rates in small open economies are on average counter-cyclical to aggregate income, they may be affected by a wide range of other factors than a country's business cycle fluctuations. Let us discuss two examples: First, there is a considerable amount of noise in exchange rates. Secondly, exchange rates are bilateral variables and also respond to shocks in the rest of the world.

Regarding the first point, a borrower that takes on local currency swaps for insurance purposes simultaneously exposes himself to noise in exchange rates. This creates a new source of risk and naturally limits the amount of foreign currency swaps he is willing to hold. Examples for such noise include currency flucutations created by arbitrary or unreliable monetary policy and fluctuations that are endogenously generated by noise traders without any corresponding changes in macroeconomic fundamentals (Jeanne and Rose, 2002). While a large number of central banks around the world have managed to make monetary policy more predictable in recent decades, the noise in exchange rates that is endogenously created by financial markets continues to be of important concern. As discussed in Bohn (1990); Jeanne (2003), if the real value of the local currency becomes too unstable, private agents can minimize the risks they are exposed to by taking on foreign instead of local currency debt, despite the procyclicality that this introduces. An example for how noisy monetary shocks limit the insurance capacity of foreign and local currency bonds is provided by section 6 of Devereux and Sutherland (2009).

The second issue we raised was that exchange rates typically also respond to shocks that occur outside the small open economy. Exchange rate fluctuations that are driven by external macroeconomic developments are detrimental to domestic agents' capacity to use local currency debt and foreign reserves for insurance purposes. This might be to some extent desirable, as it allows international investors to offload risk to domestic agents and enhances international risk sharing.[11] However, such a risk transfer is un-

[11] Indeed, we conjecture that a parallel result to the risk-sharing result of this paper holds in models of two or more large economies: large economies with counter-cyclical exchange rates could perfectly share idiosyncractic risks by holding infinite amounts of local currency debt and foreign currency reserves, since the payoff of foreign currency swaps reflects the relative position in the business cycle of the two economies (Ching and Devereux, 2000).

desirable if foreign investors are indeed risk-neutral. Furthermore, if global shocks simultaneously hit the small open economy and the rest of the world, exchange rates might will be little affected. As a result, local currency could not be used to insure against global aggregate risk.

Risk Premia on Local Currency

Finally, a fourth important assumptions underlying our main proposition is that international lenders are either risk-neutral or can perfectly diversify the small open economy's risk factors. The recent empirical literature (see e.g. Dodd and Spiegel, 2005; Burger and Warnock, 2006b) suggests that borrowers in many economies – especially emerging market economies where insurance would be most valuable – face a positive risk premium on local currency debt.[12]

While the precise theoretical reasons for this positive risk premium are not fully understood (see Burger and Warnock, 2006a), it makes it costly to buy local currency swaps for insurance purposes. It encourages foreign as opposed to local currency borrowing and can even lead to a situation where agents borrow in foreign and invest in local currency assets to earn the expected return differential. A deviation from interest rate parity therefore weakens incentives to use local currency debt for insurance purposes.

8 Conclusion

This paper analyzes the insurance properties of bonds denominated in local and foreign currency in standard small open economy models where interest rate parity holds and where exchange rates are counter-cyclical. We showed that agents who have unconstrained access to the two bonds would use local currency borrowing and holdings of foreign reserves (i.e. foreign exchange swaps) to insure against aggregate fluctuations. A fortiori, they would never take on net foreign currency debt. Their optimal portfolio would in the limit consist of an infinite amount of local currency debt and corresponding foreign currency reserves, and this portfolio would perfectly insure the economy

[12]This risk premium is the observed differential in interest rates between local and foreign currency, which is readily observable, minus the expected rate of depreciation of the local currency, which is unobservable. This unobservability implies that tests of the existence of a positive risk premium tend to have low power.

against all aggregate fluctuations. By the same token, we showed that local currency debt and foreign currency reserves could perfectly insure a small open economy against the collapse of an imperfectly credible peg.

In contrast to our theoretical findings, empirical evidence shows that many small open economies – and in particular emerging market economies – incur significant amounts of foreign currency denominated debts, which increases volatility rather than providing insurance. We discussed a number of potential relaxations of our assumptions that allow models of small open economies to describe a non-degenerate portfolio choice between local and foreign currency debt. This discussion provides some insights into how to modify standard small open economy macro models to allow for a non-degenerate analysis of questions of debt denomination and insurance.

Our findings also suggest that developing deep bond markets in local currency might offer significant insurance benefits to small open economies. Depending on how closely a country's exchange rate tracks aggregate consumption, local currency bonds may be an attractive alternative to GDP-linked foreign currency bonds. This latter point is of particular relevance for emerging market economies that are prone to crisis, since a country's exchange rate depreciates much more than its GDP declines in a typical financial crisis.

A Alternative Models of Exchange Rate Determination

The main body of the paper derives our results in a model of real exchange rates. This appendix illustrates that our assumption that the exchange rate is a strictly monotonic function of aggregate consumption holds in most other popular models of exchange rate determination. Our results in propositions 2 and 3 therefore apply equally to these classes of models.

A.1 Exchange Rate Models Based on Money Demand

Standard monetary models of the exchange rate are based on the observation that money demand is an increasing function of aggregate consumption. In response to aggregate shocks, the price level adjusts to equate real money demand to the given money supply, and the nominal exchange rate E adjusts in parallel with the price level (see e.g. Frenkel, 1976). As a result the exchange rate is counter-cyclical, i.e. it is a strictly decreasing function of aggregate consumption.

To illustrate these findings analytically, we extend the simple endowment economy of section 2 to include money and assume that the representative domestic agent values real balances M/P according to the simple utility function $u(C, M/P) = \log C + \gamma \log M/P$.[13] In addition to his income Y^ω, he is endowed with an exogenous amount of nominal money $\bar{M}$. The Lagrangian to the optimization problem of the domestic agent is

$$\mathcal{L} = u\left(C^\omega, \frac{M^\omega}{P^\omega}\right) - \lambda\left[P^\omega C^\omega + M^\omega - P^\omega Y^\omega - \bar{M}\right]$$

The agent combines the first-order conditions for optimal consumption C^ω and optimal nominal money holdings M^ω to find the equilibrium condition

$$\gamma C^\omega = \frac{M^\omega}{P^\omega} \tag{A.1}$$

Money market clearing requires that $M^\omega = \bar{M}$ in equilibrium. As a result, the price level in the economy is inversely related to aggregate consumption,

$$P^\omega = \frac{\bar{M}}{\gamma C^\omega} = e(C^\omega) \quad \text{which satisfies} \quad e'(C^\omega) < 0$$

The second equality reflects the assumption of purchasing power parity, according to which the money price of international currency (i.e. the exchange rate e) is proportional to the money price of tradable goods. It is clear that the exchange rate is a strictly decreasing function of aggregate consumption. A negative shock to aggregate income, for example, reduces real money demand; for given money supply $\bar{M}$, the domestic price level and the nominal exchange rate have to rise (i.e. depreciate) to equilibrate the market.

If we allow for local and foreign currency debt in the framework given in this section and express the agent's income Y^ω as in equation (2) to incorporate the insurance effects of foreign exchange swaps, it can be seen that the full insurance result discussed for real exchange rates earlier in the paper also applies to monetary models of the exchange rate. The proof follows the same steps as in section 3 and is available from the author upon request.[14]

[13] As shown in Feenstra (1986), note that most macroeconomic models of money demand, including models of money-in-the-utility, money search, cash-in-advance constraints, etc. are functionally equivalent. Similar results hold under price stickiness, which exacerbates exchange rate movements (see Dornbusch, 1976).

[14] In fact, the exchange rate equation above reveals how closely related the class of monetary models and real models of the exchange rate are: in section 3 the exchange rate is counter-cyclical because tradable and non-tradable goods are complements and the price of non-tradable goods rises when tradable consumption increases; here goods and real balances are complements and the price of money rises when real consumption increases.

A.2 Fiscal Models of the Exchange Rate

In fiscal models of the price level and of the exchange rate (see e.g. Ize, 1987), a government finances its spending through taxation and, if regular tax revenue is insufficient, through seigniorage. When aggregate demand is low, regular tax receipts are low and the government needs to print more money to raise funds, thereby inflating the price level and depreciating the exchange rate.

To illustrate these effects analytically, we add a government sector to the model above and assume that government needs to finance an exogenous amount of spending $\bar{G}$. This spending can be financed in two ways: (i) Government can raise taxes on the representative agent's gross expenditure C^ω, but we assume the tax rate to be capped at τ because of political economy concerns or adverse incentive effects. (ii) The gap between spending and regular tax revenue needs to be financed by printing a nominal amount ΔM of money:[15]

$$\bar{G} = \tau C^\omega + \frac{\Delta M^\omega}{P^\omega}$$

Assuming an initial money supply of M_0, market clearing implies that total nominal money demand has to equal the available supply $M_0 + \Delta M$. In equilibrium, equation (A.1) then implies that

$$\gamma C^\omega = \frac{M_0 + \Delta M^\omega}{P^\omega} = \frac{M_0}{P^\omega} + \bar{G} - \tau C^\omega$$

Solving this equation for the market-clearing aggregate price level and assuming the nominal exchange rate e moves in parallel with the price level yields

$$e(C^\omega) = P^\omega = \frac{M_0}{(\gamma + \tau)C^\omega - \bar{G}} \quad \text{which satisfies} \quad e'(C^\omega) < 0$$

For example, a negative aggregate shock that reduces C^ω entails lower tax receipts τC^ω. More revenue needs to be raised through seigniorage ΔM^ω and therefore more inflation and exchange rate depreciation follow. Again, the counter-cyclical exchange rate can be used to obtain full insurance as a limiting result, as demonstrated in section 3. A detailed proof is available upon request.

Most economies experience fiscal dominance over monetary policy only in some rare states of nature. The model outlined here assumes for simplicity that the price level is always determined by fiscal factors. However, the model constitutes a basic building block that can easily be integrated into other models of exchange rate determination, including models of exchange rate pegs that collapse in response to strong adverse shocks.

[15] To simplify our analysis and to rule out degenerate results, we impose the assumption $\tau C^\omega < \bar{G} < (\tau + \gamma)C^\omega \; \forall \omega$, that government always resorts to some amount of seigniorage, but that the spending can ultimately be financed that way.

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